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Wavelet compressed, modified Hilbert transform in the space-time discretization of the heat equation

On a finite time interval $(0,T)$, we consider the multiresolution Galerkin discretization of a modified Hilbert transform $\mathcal{H}_T$ which arises in the space-time Galerkin discretization of the linear diffusion equation. To this end, we design spline-wavelet systems in $(0,T)$, consisting of piecewise polynomials of degree ≥ 1 with sufficiently many vanishing moments, which constitute Riesz bases in the Sobolev spaces $H^s_{0,}(0,T)$. These bases provide multilevel splittings of the temporal discretization spaces into "increment" or "detail" spaces of direct sum type. Via algebraic tensor-products of these temporal multilevel discretizations with standard, hierarchic finite element spaces in the spatial domain (with standard Lagrangian FE bases), sparse space-time tensor-product spaces are obtained, which afford a substantial reduction in the number of the degrees of freedom as compared to time-marching discretizations. In addition, temporal spline-wavelet bases allow to compress certain nonlocal integrodifferential operators which appear in stable space-time variational formulations of initial-boundary value problems, such as the heat equation and the acoustic wave equation. An efficient preconditioner is proposed that affords essentially linear complexity solves of the linear system of equations which results from the full and sparse space-time Galerkin discretizations.