Stabilized FEM for Contact Problems

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Abstract

Stabilization of mixed finite element methods for saddle point problems is a well-established technique that allows one to use finite element spaces that do not satisfy the Babuška–Brezzi condition. They were introduced and analysed in 80's by Hughes, Franca, Brezzi, Pitkäranta and others. The analysis has, however, suffered from the fact that full regularity of the exact solution needs to be assumed.

In this talk, we give an overview of our recent work on stabilized FEM for contact problems. The problems are written in a mixed form, with the contact pressure acting as a Lagrange multiplier, and the stabilised formulation is derived by adding appropriately weighted residual terms to the discrete variational forms.

We show that the discrete formulation is uniformly stable and that it leads to a quasi-optimal a priori error estimate without further regularity assumptions. Moreover, we establish a posteriori estimates whose optimality is ensured by local lower bounds.

To implement the method, the discrete Lagrange multiplier can be locally eliminated, thus giving rise to a robust Nitsche-type method. We present a series of numerical results which confirm the optimality of our a posteriori estimates.

This is a joint work with Tom Gustafsson (Aalto University) and Juha Videman (Instituto Superior Técnico, University of Lisbon).

References

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