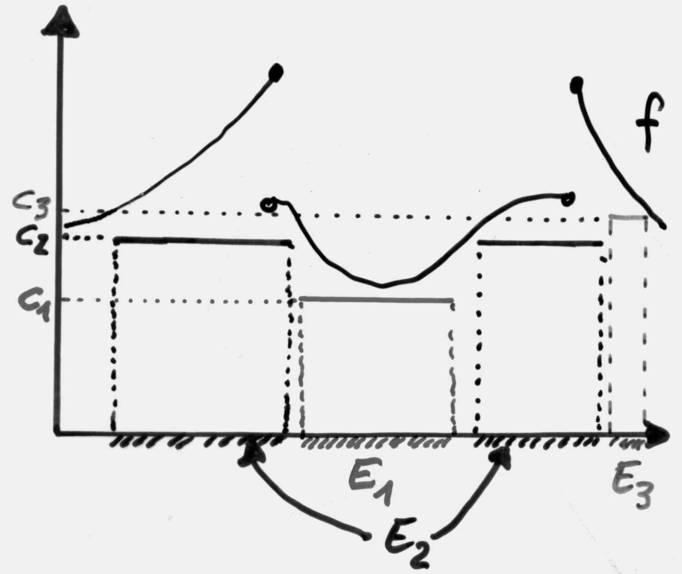
Lebesgue-Integral

f messbar, ≥ 0

$$\chi_E(x) = \begin{cases} 1, & x \in E \\ 0, & x \notin E \end{cases}$$



Approximiere f durch

$$0 \leq \sum_{j=1}^{k} c_j \chi_{E_j} \leq f,$$

E_j messbar.

$$\int dm\, f = \sup_{0 \leq \sum_{j=1}^{k} c_j \chi_{E_j} \leq f} \sum_{j=1}^{k} c_j\, m(E_j)$$

f integrierbar $:\Longleftrightarrow \int dm\, f < \infty$

f nicht ≥ 0: • $\int dm\, f := \int dm \max\{0, f(x)\}$
$\qquad - \int dm \max\{0, -f(x)\}$

• integrierbar $:\Longleftrightarrow$ beide Terme $< \infty$

<u>Eigenschaften:</u> f, g int.bar, E, F messbar

- $\int_E dm\, f := \int dm\, f \chi_E$
- $\int dm\, \{c \cdot f + d \cdot g\} = c \int dm\, f + d \int dm\, g$
- $\int_{E \cup F} dm\, f = \int_E dm\, f + \int_F dm\, f$, falls $m(E \cap F) = 0$
- Ist h messbar, so ist h intbar $\Longleftrightarrow |h|$ int.bar
- $h : [a,b] \to \mathbb{R}$ Riemann-intbar $\Rightarrow$ h intbar

Dann $\int_{[a,b]} dm\, h = \int_a^b dx\, h.$