Differentialformen

1-Formen auf $U \subset \mathbb{R}^n$:

- $\omega(x) = \omega_1(x)\,dx_1 + \dots + \omega_n(x)\,dx_n\,, \quad \omega_j : U \to \mathbb{R}$
- $$\int_\gamma \omega = \int_a^b \omega_1(\underbrace{\gamma(t)}_{x = \gamma(t)})\, \underbrace{\gamma_1'(t)\,dt}_{dx_j = \gamma_j'(t)\,dt} + \dots + \int_a^b \omega_n(\gamma(t))\,\gamma_n'(t)\,dt$$
  $\gamma : [a,b] \to U$ Kurve
- $df = \partial_{x_1} f\,dx_1 + \dots + \partial_{x_n} f\,dx_n\,, \quad f : U \to \mathbb{R}$ diffbar
- ω exakt $\Leftrightarrow$ $\omega = df$

k-Formen auf $U \subset \mathbb{R}^n$:

- $$\omega(x) = \sum_{i_1 < \dots < i_k} \omega_{i_1 \dots i_k}(x)\,dx_{i_1} \wedge \dots \wedge dx_{i_k}$$
- $\wedge$ linear, $dx_i \wedge dx_j = -\,dx_j \wedge dx_i\,, \quad dx_i \wedge dx_i = 0$
- $$d\omega = \sum_{i_1 < \dots < i_k} d\omega_{i_1 \dots i_k} \wedge dx_{i_1} \wedge \dots \wedge dx_{i_k}$$

Poincaré:
$$\left\{\begin{aligned} \nabla f = 0 &\Leftrightarrow f = c \in \mathbb{R} \\ \nabla \times E = 0 &\Leftrightarrow E = \nabla \phi \\ \nabla \cdot B = 0 &\Leftrightarrow B = \nabla \times A \end{aligned}\right\} \Longleftarrow \quad d\omega = 0 \underset{\uparrow}{\Leftrightarrow} \omega = d\eta$$

U „einfach zusammenhängend"

Stokes:
$$\left\{ \int_a^b \nabla f = f(b) - f(a),\ \int_A \nabla \cdot F = \int_{\partial A} \nu \cdot F,\ \int_M \nu \cdot (\nabla \times F) = \int_{\partial M} \tau \cdot F \right\} \Longleftarrow \int_M d\omega = \int_{\partial M} \omega$$