Integration auf UM:

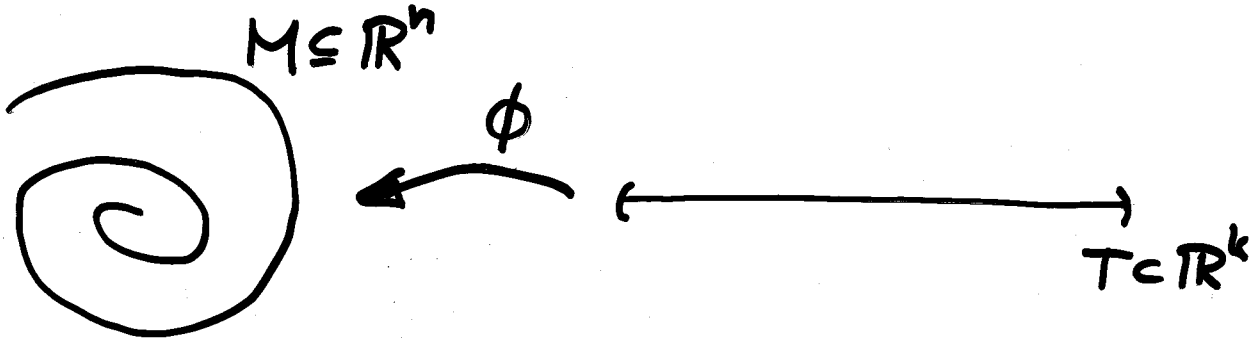
- **Fall 1:** $\phi : T \to M$ globale Karte ($\phi(T) = M$)

$$\int_M f(x)\, dS(x) := \int_T f(\phi(t)) \sqrt{\det \partial\phi(t)^T \partial\phi(t)}\, dt$$

$x = \phi(t)$

$dS(x) = \sqrt{\det \partial\phi^T \partial\phi}\, dt$

Verzerrungsfaktor



Spezialfall: $T, M \subseteq \mathbb{R}^n$ offen, ϕ C^1-Diffeo.

$$\Rightarrow \int_M f\, dx \overset{\uparrow}{=} \int_T f(\phi(t)) \, |\det \partial\phi(t)| \, dt$$

Transformationssatz (4.3)

$$|\det \partial\phi| = \sqrt{(\det \partial\phi^T)(\det \partial\phi)} = \sqrt{\det(\partial\phi^T \partial\phi)}$$

$$\Rightarrow \int_M f\, dx = \int_T f(\phi(t)) \sqrt{\det(\partial\phi^T \partial\phi)}\, dt$$

- **Fall 2:** $\varphi_j : T_j \to M$ lokale Karten

$\Rightarrow$ Addiere Beiträge der lokalen Karten