k × Münze werfen:

$X_k = \{Z,K\}^k$, $p_k : \mathcal{P}(X_k) \to [0,1]$, $p_k(\{(x_1,\dots,x_k)\}) = \frac{1}{2^k} = \frac{|\{(x_1,\dots,x_k)\}|}{2^k}$

(siehe vorher)

$f_k : X_k \to \mathbb{R}$, $\tilde{f}_{k-1} : X_k \to \mathbb{R}$, $g_k : X_k \to \mathbb{R}$.

f_k = Anzahl K in k Versuchen. **Zeige:** $\langle f_k \rangle = \frac{k}{2}$

$\tilde{f}_{k-1}$ = Anzahl K in den ersten k-1 Versuchen

$g_k = \begin{cases} 1, & \text{falls im k-ten Wurf K} \\ 0, & \text{falls im k-ten Wurf Z} \end{cases}$

$\Rightarrow f_k = \tilde{f}_{k-1} + g_k$

$\langle f_k \rangle_{X_k} \equiv \langle f_k \rangle$ (den Raum X lässt man bei $\langle \dots \rangle_X$ weg, falls er klar ist)

$$= \sum_{x=(x_1,\dots,x_k)\in X_k} p_k(\{x\}) \underbrace{f_k(x)}_{= \tilde{f}_{k-1}(x) + g_k(x)} = \sum_{x=(x_1,\dots,x_k)\in X_k} \frac{1}{2^k} \underbrace{\tilde{f}_{k-1}(x_1,\dots,x_k)}_{\substack{= \text{Anzahl K in } (x_1,\dots,x_{k-1}) \\ = f_{k-1}(x_1,\dots,x_{k-1})}} + \sum_{x=(x_1,\dots,x_k)\in X_k} \frac{1}{2^k} g_k(x_1,\dots,x_k)$$

$$= \sum_{\substack{(x_1,\dots,x_{k-1})\in X_{k-1} \\ x_k \in \{Z,K\}}} \frac{1}{2^k} f_{k-1}(x_1,\dots,x_{k-1}) + \underbrace{\sum_{(x_1,\dots,x_{k-1})\in X_{k-1}} \frac{1}{2^{k-1}}}_{=1} \underbrace{\sum_{x_k\in\{Z,K\}} \frac{1}{2} g_k(x_1,\dots,x_k)}_{= \frac{1}{2}(\underset{x_k=K}{1} + \underset{x_k=Z}{0})}$$

$$= \underbrace{\sum_{x_k\in\{Z,K\}} \frac{1}{2}}_{=1} \underbrace{\sum_{(x_1,\dots,x_{k-1})\in X_{k-1}} \frac{1}{2^{k-1}} f_{k-1}(x_1,\dots,x_{k-1})}_{= \langle f_{k-1} \rangle_{X_{k-1}}} + \frac{1}{2}$$

$$= \langle f_{k-1} \rangle_{X_{k-1}} + \frac{1}{2}$$

Induktion: $\langle f_1 \rangle_{X_1} = p(\{K\})\, f_1(K) + p(\{Z\})\, f_1(Z) = \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 = \frac{1}{2}$

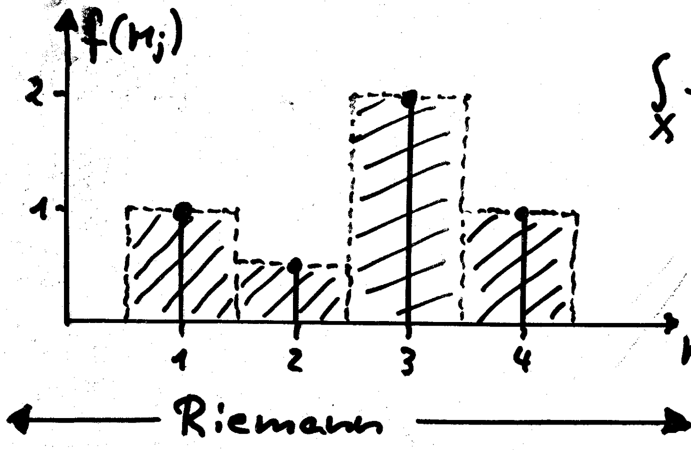
Beh: $\langle f_k \rangle_{X_k} = \frac{k}{2}$. ⌈Beweis: $k=1$ ✓ Annahme: $\langle f_{k-1} \rangle_{X_{k-1}} = \frac{k-1}{2}$

$\langle f_k \rangle_{X_k} = \langle f_{k-1} \rangle_{X_{k-1}} + \frac{1}{2} = \frac{k-1}{2} + \frac{1}{2} = \frac{k}{2}$ ✓ ⌋

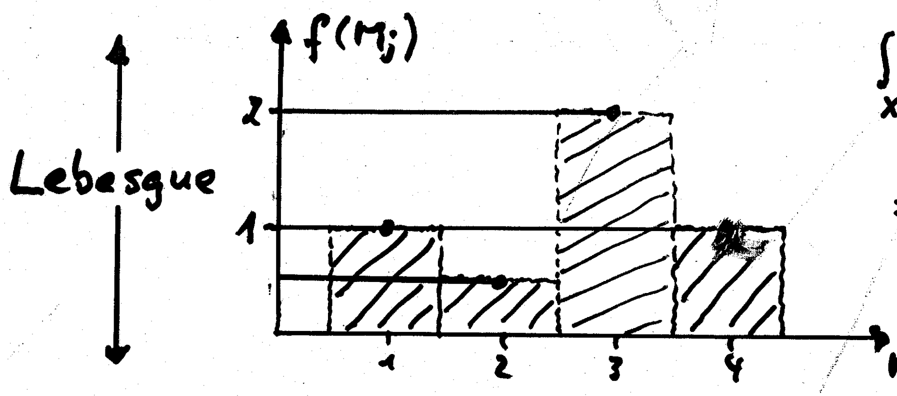
Hinweis: Diese Methode sollten Sie in 1b) entweder anwenden oder darauf verweisen. Sie können 1b) natürlich auch anders lösen...

$$X = \{ \underset{M_1}{1}, \underset{M_2}{1/2}, \underset{M_3}{2}, \underset{M_4}{1} \}, \quad \mu(\{M_j\}) = 1$$

$f : X \to \mathbb{R}$ Wert der Münze



$$\int_X f\, d\mu = \sum_{M_j \in X} f(M_j)\, \mu(\{M_j\}) = 1 \times 1 + \tfrac{1}{2} \times 1 + 2 \times 1 + 1 \times 1$$



$$\int_X f\, d\mu = \sum_{y \in \mathbb{R}} y\, \mu(f^{-1}(\{y\})) = \tfrac{1}{2} \times 1 + 1 \times 2 + 2 \times 1$$