Maxwell Mini-Symposium Analysis and its Applications, 27 February 2015

Abstracts

Pierre Degond (Imperial College)
Mathematical models of self-organization: bifurcations and derivation of macroscopic models

We consider self-organizing systems, i.e. systems consisting of a large number of interacting entities which spontaneously coordinate and achieve a collective dynamics. Such systems are ubiquitous in nature (flocks of birds, herds of sheep, crowds, ...). Their mathematical modeling poses a number of fascinating questions such as finding the conditions for the emergence of collective motion. In this talk, we will consider a simplified model first proposed by Vicsek and co-authors and consisting of self-propelled particles interacting through local alignment. We will derive its hydrodynamic limit using the recent concept of generalized collision invariant. We will also rigorously study the multiplicity and stability of its kinetic equilibria. We will illustrate our findings by numerical simulations.

Martin Dindos (Edinburgh)
The Dirichlet boundary problem for second order parabolic operators satisfying Carleson condition

This is a joint work with Sukjung Hwang. We establish L^p, $2 \leq p \leq \infty$ solvability of the Dirichlet boundary value problem for a parabolic equation $u_t - \mathrm{div}(A\nabla u) - B \cdot \nabla u = 0$ on time-varying domains with coefficient matrices $A = [a_{ij}]$ and $B = [b_i]$ that satisfy a small Carleson condition. The result is motivated by similar results for the elliptic equation $\mathrm{div}(A\nabla u) + B \cdot \nabla u = 0$ that were established in the papers Kenig, Pipher, Petermichl and myself. The result complements the papers of Hofmann and Rivera-Noriega where solvability of parabolic L^p (for some large p) Dirichlet boundary value problem for coefficients that satisfy large Carleson condition was established.

The main result says that given $p \in (2, \infty)$ there exists $C(p) > 0$ such that if the Carleson norm of coefficients is small then the Dirichlet boundary value problem for a parabolic equation is solvable in L^p.

I shall also discuss a second result (with J. Pipher and S. Petermichl) that shows that $C(p) \to \infty$ as $p \to \infty$ which is a quantitative result complementing the previous results on L^p solvability under large Carleson condition (where only existence of $p < \infty$ is established, but not how it might depend on the Carleson norm).

Ben Goddard (Edinburgh)
Complex, multiscale modelling: applications of analysis

Producing good mathematical models of real-world problems is a highly challenging task. It is very easy to produce bad models, but developing good ones generally requires a synergistic combination of physical understanding, mathematical analysis and computational techniques. After discussing what (I think) makes a good mathematical model, I'll discuss some of the challenges of modelling complex systems with multiple scales, e.g. in time or length. This will be motivated using a wide range of examples from interdisciplinary research. Whilst such problems are routinely tackled in applied sciences, most approaches pay little attention to the underlying mathematical properties or structure and the resulting models can, put politely, 'lack some desired features'. I'll describe some ways in which mathematical analysis can significantly improve the quality of these models, often whilst also reducing the computational cost. If time allows, I'll give an example of my approach to such problems, which combines mathematical modelling with rigorous analysis and numerics.