ABSTRACTS FOR WORKSHOP ON *MAGNITUDE 2019: ANALYSIS, CATEGORY THEORY, APPLICATIONS* ICMS, EDINBURGH 4-5/7–2019

Theme: Magnitude

Tom Leinster (University of Edinburgh),
An overview of magnitude
This workshop brings together researchers with interests in various different aspects of magnitude, some of which may appear to have little overlap with others. In this introductory talk, I will paint a broad-brush picture of the wide world of magnitude as I see it, in the hope that participants will gain further understanding of how their own work relates to other people's. I will include some of the categorical background and some of the uses of magnitude in contexts other than metric spaces, as well as an introduction to the aspect of magnitude that has probably received the most attention, the magnitude of metric spaces.

Mark Meckes (Case Western Reserve),
The magnitude of an infinite metric space
I will start with an overview of the different approaches we have to extending magnitude from finite metric spaces to infinite metric spaces. I will then focus on the settings in which magnitude of infinite spaces is best-understood, namely $\mathbb{R}^n$ with the Euclidean or taxicab metric, and the connections that arise there between magnitude and other, more classical ways of quantifying the "size" of a set. Throughout the talk I will emphasize the basic things we don't yet know about the definition or behavior of magnitude (but would like to!).

Anthony Carbery (University of Edinburgh),
On the magnitudes of some compact sets in Euclidean spaces – focusing on the boundary
Joint work with J.A. Barceló. We study the magnitudes of compact sets in Euclidean spaces. We describe the leading-term asymptotics of the magnitude of such sets in the small and large-scale regimes. We then consider the magnitudes of compact convex sets with nonempty interior in Euclidean spaces of odd dimension, and relate them to the boundary behaviour of solutions to certain naturally associated higher order elliptic boundary value problems in exterior domains. We carry out calculations leading to an algorithm for explicit evaluation of the magnitudes of balls, and this establishes the convex magnitude conjecture of Leinster and Willerton in the special case of balls in dimension three. In general we show that the magnitude of an odd-dimensional ball is a rational function of its radius.

Simon Willerton (University of Sheffield),
Magnitude of odd balls
Explicitly calculating the magnitude function of infinite subsets of euclidean space is not something we have been very successful at doing. For convex subsets, Tom Leinster and I had conjectured a polynomial formula involving classical invariants of size; Juan Antonio Barcelo and Tony Carbery showed this conjecture was true for

the three-ball but failed for higher dimensional balls, where in fact the magnitude functions are rational functions.

In this talk I will tell the story of how I obtained (rather lovely!) explicit formulas for these rational functions. As with many magnitude stories, it involves characters from disparate parts of mathematics, in this case these characters include certain new identities for special functions – which wouldn't have looked out of place in the nineteenth century – and somewhat more contemporary enumerative combniatorics which allow you to calculate the coefficients in the rational functions by counting. I will also endeavour to explain how these formulas feed into other work.

Theme: Analysis of magnitude

Anders and Jana Björn (University of Linköping),
Capacities: What are they and what are they good for
There are many different types of capacities but most of them are associated in one way or another with Sobolev spaces, and thus also differential equations. We will define and discuss so-called Sobolev and variational capacities.

This can be done in various types of generality, and we will consider capacities associated with the higher-order Sobolev spaces $W^{k,p}(\mathbb{R}^n)$, the weighted first-order Sobolev spaces $W^{1,p}(\mathbb{R}^n, d\mu)$ as well as first-order Sobolev spaces on metric spaces.

Capacities are a finer notion than measure and one can get more precise results about functions and their properties by using capacities rather than measures. We will give several examples of this.

Ritva Hurri-Syrjänen (University of Helsinki),
On capacity estimates
We will review some Sobolev capacity estimates.

Magnus Goffeng (University of Gothenburg and Chalmers),
The magnitude function of domains, part I
I will present some (fairly) recent results on the magnitude function for compact metric spaces of geometric origin (i.e. domains in Euclidean space or manifolds). One of the results states that the magnitude recovers geometric invariants such as volume and certain integrals of curvatures. Based on joint work with Heiko Gimperlein and Nikoletta Louca.

Nikoletta Louca (Heriot-Watt University),
The magnitude function of domains, part II
In this talk we will see some further properties of the magnitude function. We further discuss an approach to calculating the magnitude of a domain using integral equations as opposed to reformulating to a boundary value problem. This is based on joint work with Heiko Gimperlein and Magnus Goffeng.

Emily Roff (University of Edinburgh),
The maximum diversity of a compact metric space
In this talk, I will introduce a family $(D_q)_{q\in[-\infty,\infty]}$ of distance-sensitive indices of the spread, or entropy, of a probability measure on a compact metric space. These are referred to as diversity indices, because their definition in a finite setting - in work by Christina Cobbold and Tom Leinster - was first motivated by problems in mathematical ecology.

In general, different choices for the parameter q lead to different assessments of the diversity of a probability measure. Nevertheless, it can be shown that when it comes to maximising diversity, there is consensus: every compact metric space admits a probability measure which maximises the order-q diversity for every non-negative value of q at once. Moreover, the diversity of any D_q-maximising measure is independent of the choice of q. This yields a real-valued invariant of compact metric spaces, maximum diversity, which is closely related both to magnitude and, via a result of Mark Meckes, to Minkowski dimension. Studying diversity-maximising measures on a compact space as its metric is scaled, we also obtain something like a canonical uniform measure on any compact metric space.

This is joint work with Tom Leinster, generalising results due to Leinster and Meckes.

Theme: Magnitude homology

Michael Shulman (University of San Diego),
Introduction to magnitude homology
The magnitude of an enriched category is a general construction which specializes not only to the magnitude of a metric space but also to the Euler characteristic of (the nerve of) an ordinary category. Such Euler characteristics have many other equivalent descriptions, but one of the most important is as an alternating sum of the ranks of the *homology groups*, a set of algebraic invariants that carry "dimension-wise" information about the topology of a space, and additionally also make sense for infinite spaces when the Euler characteristic does not. Thus, it is natural to ask whether magnitude of enriched categories, and specifically of metric spaces, can also be obtained in such a way from a "magnitude homology" theory that carries additional geometric information and makes sense for infinite categories and spaces.

Hepworth and Willerton constructed such a magnitude homology theory for the special case of graphs (regarded as metric spaces on their vertices with integer path lengths as distances). I will describe a general construction of magnitude homology for enriched categories (joint work with Leinster) that includes the Hepworth-Willerton theory for graphs, as well as ordinary homology for ordinary categories, and specializes to a new magnitude homology theory for arbitrary metric spaces that carries information about convexity and geodesy. Interestingly, extracting the magnitude from the magnitude homology turns out to involve subtle questions of convergence, the solution to which seems to require viewing magnitude "algebraically" (e.g. as a formal rational function or formal power series) rather than "analytically" (e.g. as a partial function on real numbers).

Nina Otter (UCLA),
Magnitude meets persistence. What happens after?
In this talk I will explain how magnitude homology is related to persistent homology, and then discuss some open questions and work in progress. In particular, I will discuss how a generalised version of this relationship might help in defining an Euler characteristic for multiparameter persistent homology. This talk is partly based on the preprint https://arxiv.org/abs/1807.01540.

Dejan Govc (University of Aberdeen),
Rips magnitude
As shown by Hepworth and Willerton in the case of graphs and later more generally by Leinster and Shulman for enriched categories, magnitude can be understood as the graded Euler characteristic of a certain homology theory. We examine an analogous invariant for persistent homology, called Rips magnitude, which arises as a filtered Euler characteristic of persistent homology. In the talk I will describe some

of its basic properties and examine its asymptotic behaviour in the case of finite subsets of the circle, using a result of Adamaszek. This is joint work with Richard Hepworth.

Kiyonori Gomi (Shinshu University),
Magnitude homology of geodesic space
The magnitude homology of a metric space is a categorification of the magnitude. The definition of the magnitude homology is easy, but its calculation is not so easy in general. I will talk about a theorem which completely describes the magnitude homology of geodesic metric spaces which satisfy a certain non-branching assumption. Examples of such metric spaces are connected and complete Riemannian manifolds. By the theorem, one can describe the magnitude homology explicitly, provided all the information of geodesics.

Masahiko Yoshinaga (Hokkaido University),
A threshold of length in magnitude homology
The magnitude homology of a metric space is a graded module with two parameters: the degree of homology and the length of chains. In this talk, we will discuss a threshold (the inf of sizes of 4-cuts) for the length. In particular, we will see that, if the length is below the threshold, the magnitude homology can be completely described by order complexes of certain posets, which enables us to compute explicitly several examples. (Joint work with R. Kaneta.)

Bino Nolting (University of Bremen),
Recovery of piecewise linear closed curves as subspaces of $\mathbb{R}^2$
Several results draw connections between discrete metric spaces and their magnitude homology groups. Hepworth introduced a method to recover dis- crete metric spaces from the magnitude cohomology ring structure. Inspired by his paper, we will tackle the class of piecewise linear closed curves in $\mathbb{R}^2$, which is a class of nondiscrete metric spaces, and show that such a space is determined by the magnitude cohomology ring. We will operate on a subset of the 0th magnitude cohomology group that corresponds to the set of points in the space. On these cohomology classes, we define the magnitude distance, a distance measure that often coincides with the actual distance between corre- sponding points. In other cases, the magnitude distance will be used to detect convergences of sequences, which will help to recover the topology of our space.

Yasuhiko Asao (University of Tokyo),
Magnitude homology of $CAT(k)$***-spaces***
We study magnitude homology of geodesic metric spacs with an upper curvature bound, including $CAT(k)$-spaces. We prove a vanishing theorem of magnitude homology of them. As a corollary, we obtain a slogan “magnitude homology measures uniqueness of geodesics”.

Richard Hepworth (University of Aberdeen),
Magnitude homology: Opinions and speculations
I will give an opinionated discussion of the the present state of magnitude homology of metric spaces, and some speculation about future directions.