Magnitude meets persistence. What happens after?

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Workshop on magnitude

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A quick tour of persistent homology

What is persistent homology?

Persistent homology is a method from algebraic topology used to study **topological features** of **data**.

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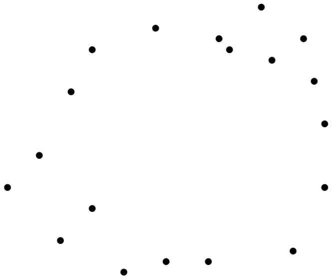
Persistent homology is a method from algebraic topology used to study **topological features** of **data**.

- **Topological features**: components, holes, voids, etc.
- **Data**: e.g., a finite metric space (X, d), often called a point cloud, a weighted undirected network, a grey-scale digital image, etc.

Persistent homology

Motivation

What is the topology of this set X of points in $\mathbb{R}^2$?



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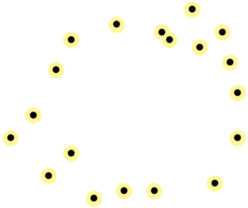
If ϵ is too small:



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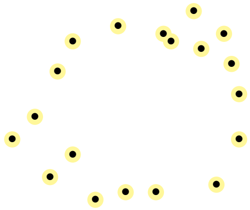
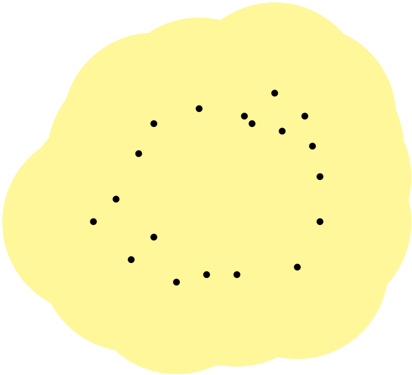


If ϵ is too large:



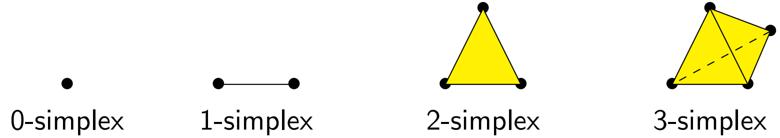
Solution: Consider all possible values for ϵ and obtain a nested sequence of spaces

$$X_{\epsilon_1} \subseteq \cdots \subseteq X_{\epsilon_n}, \text{ for } \epsilon_1 \leq \cdots \leq \epsilon_n.$$

Now study topological features of these spaces and how they evolve across the filtration.

Simplicial complexes

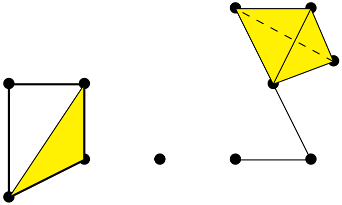
A k-**simplex** is the convex hull of $k+1$ linearly independent points in Euclidean space, e.g.:



A k-simplex is completely determined by its $k+1$ vertices. A **simplex** is a k-simplex for some k.

Simplicial complexes

A simplicial complex is built from simplices:



Homology of a simplicial complex

Given a simplicial complex K:

- the p**th simplicial homology** of K with coefficients in a field $\mathbb{K}$ is a $\mathbb{K}$-vector space $H_p(K)$
- The dimension of $H_p(K)$ is the p**th Betti number of** K**, denoted by** $\beta_p(K)$**.**

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Betti numbers give a count of:

- $p = 0$: connected components
- $p = 1$: holes
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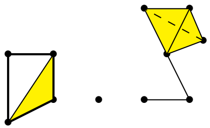

$$\beta_0 = 3$$
$$\beta_1 = 1$$
$$\beta_n = 0 \text{ for all } n \geq 2.$$

Functoriality of homology

A map of simplicial complexes $f : K \to K'$ induces a map $H_p(f): H_p(K) \to H_p(K')$ on the homology vector spaces.

[1] G. Carlsson, Topology and data, Bulletin of the AMS, 2009

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> *Functoriality has proven itself to be a powerful tool in the development of various parts of mathematics, such as Galois theory within algebra, the theory of Fourier series within harmonic analysis, and the applicaton of algebraic topology to fixed point questions in topology. We argue that [..] it has a role to play in the study of point cloud data as well.* [1]

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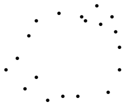
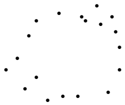
From a finite metric space to filtered simplicial complexes

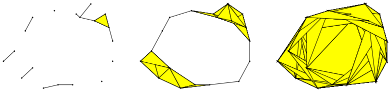
Point cloud X



From a finite metric space to filtered simplicial complexes

Point cloud X $\longrightarrow$ Filtered simplicial complex

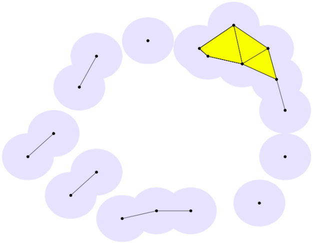




Example of simplicial complex used in persistent homology

Cech complex

$$\mathrm{C}_\epsilon(X) = \left\{\sigma \subset X \mid \bigcap_{x\in\sigma} B_x(\epsilon) \neq \emptyset\right\}$$

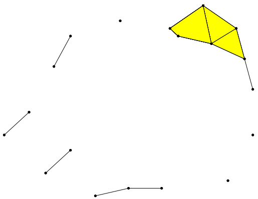


- Nerve theorem:
  $|\mathrm{C}_\epsilon(X)| \simeq \bigcup_{x\in X} B_x(\epsilon)$

Example of simplicial complex used in persistent homology

Vietoris–Rips complex

$$\mathrm{VR}_\epsilon(X) = \{\sigma \subset X \mid \forall x, y \in \sigma\colon\ \mathrm{d}(x, y) \leq 2\epsilon\}$$

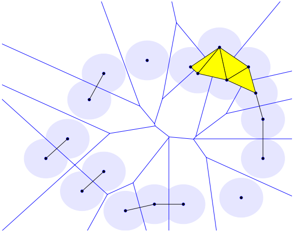


- Nerve theorem does not hold
- For all $\epsilon \geq 0$ we have $\mathrm{C}_\epsilon(X) \subset \mathrm{VR}_\epsilon(X) \subset \mathrm{C}_{\sqrt{2}\epsilon}(X)$.

Example of simplicial complex used in persistent homology

α complex

$$\alpha_\epsilon(X) = \left\{\sigma \subset X \mid \bigcap_{x\in\sigma} B_x(\epsilon) \cap V_x \neq \emptyset\right\}$$



- Nerve theorem: $|\alpha_\epsilon(X)| \simeq \bigcup_{x\in X} B_x(\epsilon)$
- For all $\epsilon \geq 0$ we have $\alpha_\epsilon(X) \subset \mathrm{C}_\epsilon(X) \subset \mathrm{VR}_\epsilon(X)$.

Persistent homology

Given a filtered simplicial complex $(K, \{K_{\epsilon_i}\}_{i=1}^{n})$, apply pth simplicial homology:

$$H_p(K_{\epsilon_1}) \xrightarrow{f_{1,2}} H_p(K_{\epsilon_2}) \xrightarrow{f_{2,3}} \dots \xrightarrow{f_{n-1,n}} H_p(K_{\epsilon_n}).$$

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More precisely, we obtain a tuple $(\{H_p(K_{\epsilon_i})\}_{i=1}^n, \{f_{i,j}\}_{i\leq j})$ such that $f_{k,j} \circ f_{i,k} = f_{i,j}$ for all $i \leq k \leq j$.

This is the p**th persistent homology** of $(K, \{K_{\epsilon_i}\}_{i=1}^n)$.

Persistence modules

In general,

- a sequence $\{M_i\}_{i\in\mathbb{N}}$ of $\mathbb{K}$-vector spaces
- a collection $\{f_{i,j}\colon M_i \longrightarrow M_j\}_{i\leq j}$ of linear maps such that $f_{k,j}\circ f_{i,k} = f_{i,j}$ for all $i \leq k \leq j$

is called a **persistence module**.

What kind of object is this?

Recall: The ring $\mathbb{K}[x]$ is $\mathbb{N}$-graded: $\mathbb{K}[x] = \bigoplus_{i\in\mathbb{N}} \mathbb{K}x^i$.

An $\mathbb{N}$-graded module M over $\mathbb{K}[x]$ is a module over $\mathbb{K}[x]$ such that $M = \bigoplus_{i\in\mathbb{N}} M_i$ and $x^j M_i \subset M_{i+j}$ for all i, j.

Correspondence theorem

Theorem (Carlsson, Zomorodian, 2005[2])

There is an isomorphism of categories between the category of persistence modules of finite type and the category of finitely generated $\mathbb{N}$*-graded modules over* $\mathbb{K}[x]$.

[2] G. Carlsson, A. Zomorodian, Computing persistent homology, Discrete & Computational Geometry, 2005

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$$\left(\{M_i\}_{i\in\mathbb{N}}, \{f_{i,j}\colon M_i \to M_j\}_{i\leq j}\right) \mapsto \bigoplus_{i\in\mathbb{N}} M_i \text{ with action of } x^j \text{ on } M_i$$

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$$\left(\{M_i\}_{i\in\mathbb{N}}, \left\{x^{j-i}\colon M_i \to M_j\right\}_{i\leq j}\right) \leftarrow\!\shortmid M = \bigoplus_{i\in\mathbb{N}} M_i \text{ graded module}$$

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Structure theorem for f.g. graded modules over a PID

Theorem (Webb 1985[3])

For any finitely generated $\mathbb{N}$-graded module M over $\mathbb{K}[x]$:

$$M \cong \left(\bigoplus_{i=1}^{n} x^{\alpha_i}\mathbb{K}[x] \right) \oplus \left(\bigoplus_{j=1}^{m} x^{\beta_j}\mathbb{K}[x]/x^{\beta_j+\gamma_j} \right) .$$

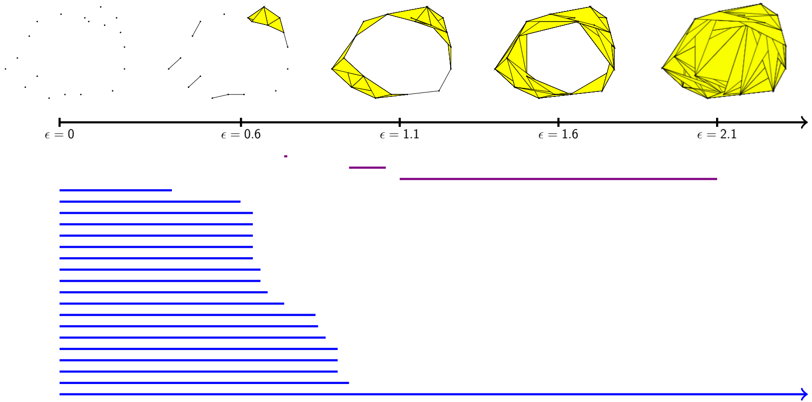
This gives:

- n infinite intervals $[\alpha_i, \infty)$ for $i = 1, \dots r$
- m finite intervals $[\beta_j, \beta_j + \gamma_j)$ for $j = 1, \dots, m$.

This collection of intervals is called **barcode**, and it is a complete invariant for persistence modules.

[3] C. Webb, *Decomposition of graded modules*, Proceedings of the AMS, 1985

Examples of barcode



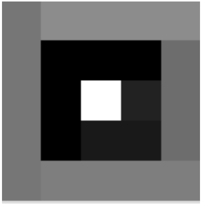
Example of Barcode

Applications of PH

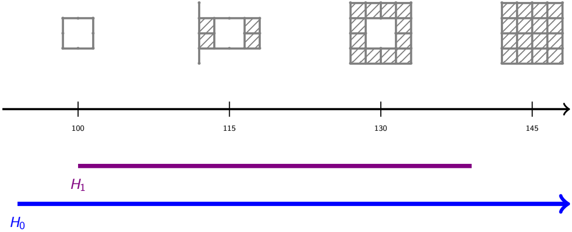
Persistent homology can be applied to, e.g.:

1. Finite metric spaces
2. Undirected weighted networks
3. Grey-scale digital images

PH to study grey-scale images

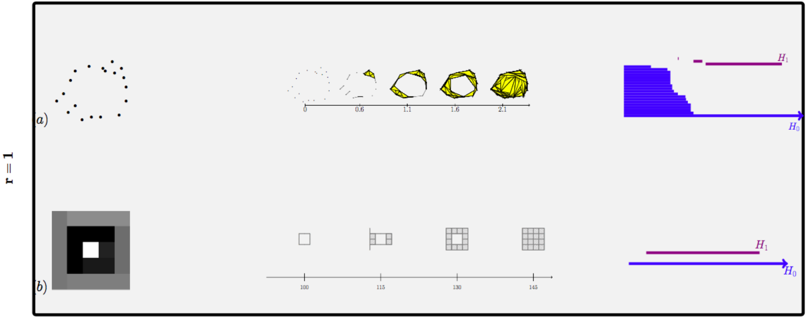


$$G = \begin{pmatrix} 115 & 119 & 119 & 119 & 119 \\ 115 & 94 & 94 & 94 & 114 \\ 115 & 94 & 139 & 100 & 114 \\ 115 & 94 & 99 & 99 & 114 \\ 115 & 117 & 117 & 117 & 117 \end{pmatrix}$$



Pipeline for PH computation

data depending on r parameters ⇨ r-filtered space ⇨ algebraic invariants



Questions

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> *(8) Magnitude homology only "notices" whether the triangle inequality is a strict equality or not. Is there a "blurred" version that notices "approximate equalities"?*
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> *... blurred magnitude homology is the persistent homology of the enriched nerve!*

Set-up

Given a metric space (X, d), we are interested in studying functors

$$CS(X) = \left([0,\infty)^{\mathrm{op}} \xrightarrow{S(X)} \mathrm{sSet} \longrightarrow ch_{\mathrm{Ab}} \right) ,$$

where for each ϵ we have that $S(X)(\epsilon)$ is a simplicial set.

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Example

The **enriched nerve of** X *is the simplicial set with set of n-simplices given by:*

$$N(X)(\epsilon)_n = \left\{ (x_0, \ldots, x_n) \mid x_i \in X, \text{ and } \sum_{i=0}^{n-1} d(x_i, x_{i+1}) \leq \epsilon) \right\}.$$

Magnitude homology chain complex

$$B(X)_n = \bigoplus_{l \in [0,\infty)} \mathbb{Z}\left[\{(x_0, \ldots, x_n) \mid \sum_{i=0}^{n} d(x_i, x_{i+1}) = l\}\right].$$

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with $d_n \colon B(X)_n \to B(X)_{n-1}$ given by the alternating sum of maps d_n^i, defined as follows for all $1 \leq i \leq n-1$:

$$d_n^i(x_0, \ldots, x_n) = \begin{cases} (x_0, \ldots, x_{i-1}, x_{i+1}, \ldots x_n), & \text{if } d(x_{i-1}, x_i) + d(x_i, x_{i+1}) \\ & \quad = d(x_{i-1}, x_{i+1}) \\ 0, & \text{otherwise} \end{cases}$$

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while for $i = 0$ we have

$$d_n^0(x_0,\dots,x_n) = \begin{cases} (x_1,x_2,\dots x_n), & \text{if } d(x_0,x_1) = 0 \\ 0, & \text{otherwise} \end{cases}$$

and similarly for $i = n$.

Equivalent definition of magnitude homology

Consider the functor of coefficients

$$A_\epsilon \colon [0,\infty) \to \mathrm{Ab}$$

$$l \mapsto \begin{cases} \mathbb{Z}, \text{if } l = \epsilon \\ 0, \text{otherwise.} \end{cases}$$

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Define the functor

$$CN(X) \otimes_{[0,\infty)} A_- \colon [0,\infty)^{\mathrm{op}} \to \mathrm{ch}_{\mathrm{Ab}}$$
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Then we have:

Proposition

$CN(X) \otimes_{[0,\infty)} A_-$ and the magnitude chain complex $B(X)$ are isomorphic.

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Magnitude homology forgets the information given by inclusion maps (what gives "persistence")!

Blurred magnitude homology

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Blurred magnitude homology

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Definition

Blurred magnitude homology is the homology of

$$\begin{aligned} C(N(X)) \otimes_{[0,\infty)} A_{[0,-]} \colon [0,\infty)^{\mathrm{op}} &\to \mathrm{ch}_{\mathrm{Ab}} \\ \epsilon &\mapsto C(N(X)) \otimes_{[0,\infty)} A_{[0,\epsilon]} . \end{aligned}$$

Blurred magnitude homology

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$$\epsilon \mapsto C(N(X)) \otimes_{[0,\infty)} A_{[0,\epsilon]}\,.$$

Proposition (NO 2018)

$C(N(X)) \otimes_{[0,\infty)} A_{[0,-]}$ and $CN(X)$ are isomorphic. In particular, blurred magnitude homology is the persistent homology of the enriched nerve.

Behaviour in the limit

Let $V(X)(\epsilon)$ be the **Vietoris–Rips** simplicial set:

$$V(X)(\epsilon)_n = \{(x_0, \ldots, x_n) \mid x_i \in X, \text{ and } d(x_i, x_j) \leq \epsilon \text{ for all } i, j\} \ .$$

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The **Vietoris homology** of X is

$$\lim_{\epsilon \to 0} H_k(CV(X)(\epsilon))\,.$$

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Theorem (NO 18)

For any metric space (X, d), we have:

$$\lim_{\epsilon \to 0} H_k(CN(X) \otimes_{[0,\infty)} A_{[0,\epsilon]}) \cong \lim_{\epsilon \to 0} H_k(CV(X)(\epsilon))$$

and

$$\lim_{\epsilon \to 0} H_k(CN(X) \otimes_{[0,\infty)} A_{\epsilon}) \cong 0\,.$$

Behaviour in the limit

Let $V(X)(\epsilon)$ be the **Vietoris–Rips** simplicial set:

$$V(X)(\epsilon)_n = \{(x_0, \ldots, x_n) \mid x_i \in X, \text{ and } d(x_i, x_j) \leq \epsilon \text{ for all } i, j\} \ .$$

The **Vietoris homology** of X is

$$\lim_{\epsilon \to 0} H_k(CV(X)(\epsilon))\,.$$

Theorem (NO 18)

For any metric space (X, d), we have:

$$\lim_{\epsilon \to 0} H_k(CN(X) \otimes_{[0,\infty)} A_{[0,\epsilon]}) \cong \lim_{\epsilon \to 0} H_k(CV(X)(\epsilon))$$

and

$$\lim_{\epsilon \to 0} H_k(CN(X) \otimes_{[0,\infty)} A_{\epsilon}) \cong 0\,.$$

That is, in the limit:

- blurred magnitude homology is Vietoris homology
- ordinary magnitude homology is trivial.

Some speculations. . .

Could blurred magnitude homology. . .

- help in categorifying magnitude for arbitrary metric spaces?

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Could blurred magnitude homology. . .

- help in categorifying magnitude for arbitrary metric spaces?
- give insight into the convex magnitude conjecture?