Maxwell Institute Graduate School on Evolution Equations, 16 – 18 September 2015

Abstracts

Short courses

Arnulf Jentzen (ETH Zürich)
An introduction to the numerical approximation of nonlinear stochastic evolution equations

This mini course gives an introduction to the research on numerical approximations of nonlinear stochastic evolution equations (SEEs) including nonlinear stochastic ordinary differential equations (SODEs) and nonlinear stochastic partial differential equations (SPDEs) as special cases. Nonlinear SEEs appear, for example, in fundamental models from financial engineering, neurobiology, and quantum field theory. In particular, we illustrate in this mini course how nonlinear SEEs are day after day used in the financial engineering industry to estimate prices of financial derivatives. We also link some of the considered financial engineering models to market data from derivative exchanges. The nonlinearities appearing in the SEEs from applications often fail to be globally Lipschitz continuous. A key topic of this minicourse is therefore the numerical approximation of SEEs with non-globally Lipschitz continuous continuous nonlinearities. In particular, we will reveal that there are some SEEs with non-globally Lipschitz continuous continuous nonlinearities which can be solved approximatively by classical and well-known approximation methods, we reveal that there are some SEEs with non-globally Lipschitz continuous continuous nonlinearities which can not be solved by classical approximation methods but by new appropriately modified numerical approximation methods, and we reveal that there are some SEEs with non-globally Lipschitz continuous continuous nonlinearities which can, roughly speaking by any time-discrete numerical approximation method, not be solved approximatively in a reasonable computational time. Some of the results in this mini course are based on joints works with Sonja Cox (University of Amsterdam), Martin Hairer (University of Warwick), Martin Hutzenthaler (University of Duisburg-Essen), Peter E. Kloeden (Huazhong University of Science & Technology), Ryan Kurniawan (ETH Zurich), Marco Noll (Frankfurt University), Thomas Müller-Gronbach (University of Passau), Primoz Pusnik (ETH Zurich), Xiaojie Wang (Central South University), and Larissa Yaroslavtseva (University of Passau).

Alessandra Lunardi (Parma)
Kolmogorov operators and invariant measures

I will describe a class of second order elliptic differential operators with unbounded coefficients in R^n, and the associated semigroups $T(t)$. The prototype is the Ornstein- Uhlenbeck operator, $Lu(x) = \Delta u(x) - < x, \nabla u(x) >$. Such operators do not enjoy nice properties in L^p spaces with respect to the Lebesgue measure. A better L^p setting is available if an invariant measure m exists, in which case $T(t)$ is a contraction in $L^p(R^n, m)$. Smoothing and summability improving properties of $T(t)$, and asymptotic behavior results are also available under reasonable assumptions. In the nonautonomous case semigroups are replaced by evolution operators and invariant measures by appropriate family of measures depending explicitly on t; however some of the good properties of the autonomous case still hold.

Lectures by senior participants

Ben Leimkuhler (Edinburgh)
Heat bath models for nonlinear partial differential equations

Thermal bath coupling techniques of molecular dynamics are applied to partial differential equations. Working from a semi-discrete (Fourier mode) formulation for the Burgers or KdV equation, we introduce auxiliary variables and stochastic perturbations in order to drive the system to sample a target ensemble which may be a Gibbs state or, more generally, any smooth distribution defined on a constraint manifold. We examine the ergodicity of approaches based on coupling of the heat bath to the high wave numbers, with the goal of controlling the ensemble through the fast modes. Our main observation is that convergence to the invariant distribution can be achieved by thermostatting just the highest wave numbers, while the dynamical processes in the low modes are little affected by such a thermostat.

Kevin Painter (Heriot–Watt)
Navigating the flow.

Navigation is a challenge for many animals, yet water and airborne species face the additional hurdle presented by the surrounding flow, from persistent currents to unpredictable storms. Hydrodynamic models are capable of simulating such dynamics and have provided the impetus for a variety of individual-based models, in which particle-sized individuals are immersed into a flowing medium. While these models can yield insights into the impact of currents on population distributions from fish eggs to large organisms, their computational demands and intractability reduces their capacity to generate the broader, less parameter specific insights of traditional continuous models. In this paper we formulate an individual-based model for navigation within a flowing field and apply scaling to obtain its corresponding macroscopic and continuous model. We apply it to various movement classes, from drifters that simply go with the flow to navigators that respond to environmental orienteering cues. The utility of the model is demonstrated via its application to "homing" problems and, in particular, the navigation of the marine green turtle Chelonia mydas to Ascension Island.

Student talks

Mate Gerencser
Degenerate SPDEs and their numerical approximations

Stochastic PDEs with possibly degenerating leading operators are considered. We derive existence and uniqueness of solutions in W_p^m spaces and extend the result to vector-valued equations. We also discuss numerical schemes for such equations and provide error estimates for localization and space and time discretization. The talk is based on joint works with I. Gyöngy and N. V. Krylov.

Oluwaseun Lijoka
Trefftz space-time discontinuous Galerkin method for the wave equation in time domain

We present a new time-space discontinuous Galerkin method for one field wave equation which utilizes special Trefftz-type basis functions. The method is motivated by the class of Interior Penalty DG (IPDG), together with the classical work of Hulbert and Hughes. The existence and uniqueness of the scheme is proved via energy argument together with a special continuity of the bilinear form. Numerical experiments highlighting the performance of the method compared with standard polynomial spaces are presented.

Eleni Moraki
Modelling Cell-Cell Adhesion

Cell adhesion is a crucial biological process of which individual cells are joining together to form tissues, organs and thus, living organisms. This biological procedure includes cell-cell adhesion and cell-extracellular matrix adhesion and plays an important role in many biological processes (i.e. embryogenesis, wound healing, cancer). In view of the fact that cell adhesion is a broad area, we focus on one of its forms and consider only how cells adhere to each other, namely cell-to-cell adhesion. Since cells do not simply "stick" together but rather through some very complex mechanisms, the understanding of this procedure becomes of a great significance. In an attempt to understand better the significant role of cell-cell adhesion, we take into account two different categories of mathematical modelling, the individual-based and the continuum models. We will discuss existing models in the literature: 2 individual based models (a stochastic cellular automaton model and the cellular Potts model) and a continuum model in a Partial Integro-Differential form.

Michele Ruggeri
Coupling and numerical integration of the Landau-Lifshitz-Gilbert equation

The nonlinear Landau-Lifshitz-Gilbert equation (LLG) models the dynamics of the magnetization field in ferromagnetic materials. Numerical challenges arise from strong nonlinearities, a nonconvex pointwise constraint which enforces length preservation, and the possible nonlinear coupling to other PDEs. In our talk, we discuss numerical integrators based on lowest-order FEM in space that are proven to be unconditionally convergent towards a weak solution of the problem. Emphasis is put on an effective numerical treatment, where the time-marching schemes decouples the numerical integration of the LLG equation and the coupled equation. The talk is based on joint work with Dirk Praetorius (TU Wien).

Michael Tsardakas
Spatiotemporal modelling of criminal activity

We examine a variety of ODE systems that model the evolution of the number of serious and minor criminals, fitting parameters from actual crime data . We also propose a diffusion-based PDE model that describes the spatiotemporal evolution of criminal activity in a city. The model successfully replicates the usual patterns of crime distribution in a city, such as the formation and persistence in time of crime *hotspots*.