Navigating the flow: Animal navigation in fluid environments

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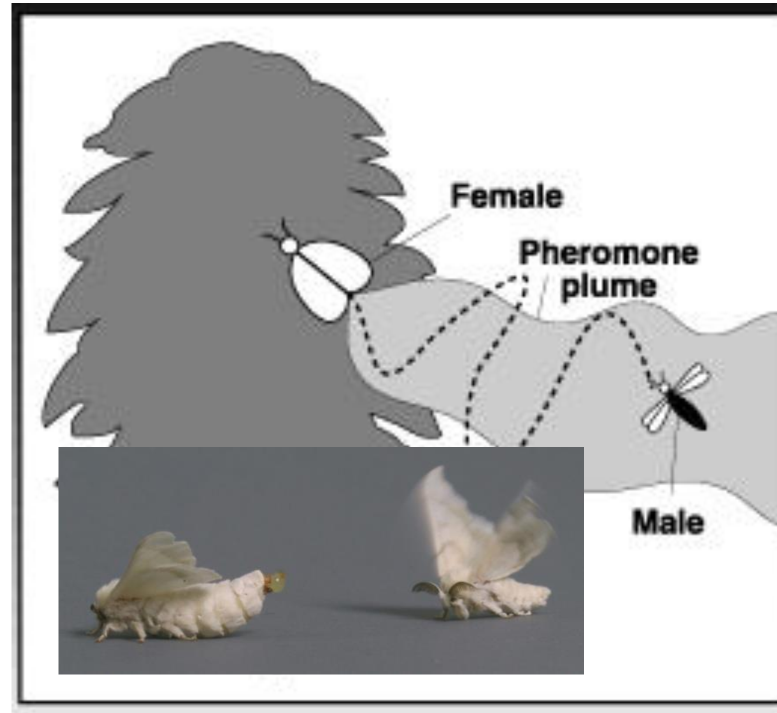
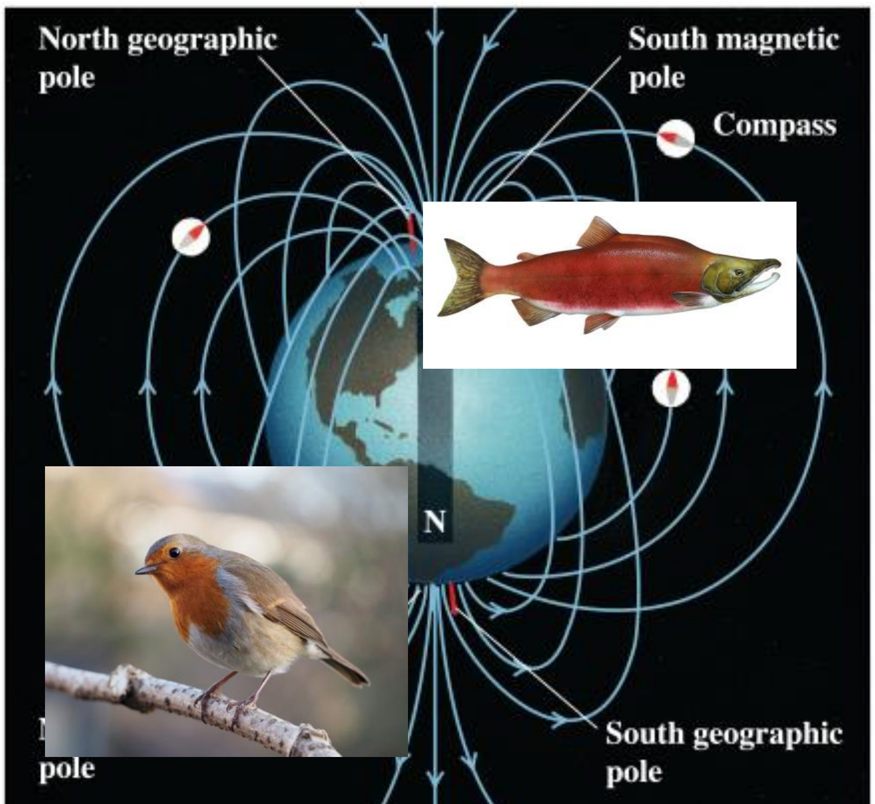
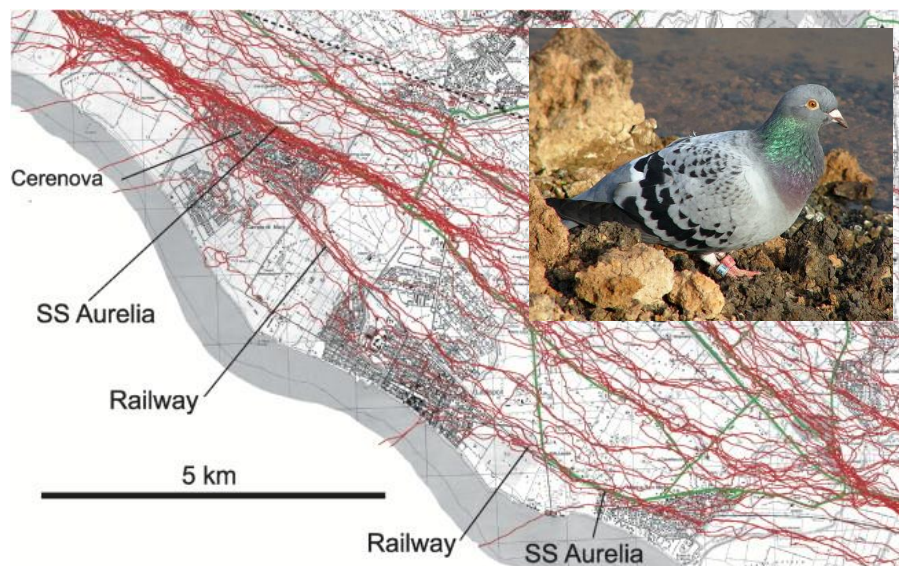

Female
Pheromone plume
Male


North geographic pole
South magnetic pole
Compass
N
pole
South geographic pole


Cerenova
SS Aurelia
Railway
5 km
Railway
SS Aurelia

But: airborne and acquatic organisms must deal with significant and complex flows...

Talk outline

- A classic navigating problem: homing of green turtles (*Chelonia mydas*) to Ascension Island
- Data and modelling
- A multiscale approach: from an IBM to continuous models
- Application to homing.

Green Turtle Navigation

Atlantic adult green turtles return to nest at hatchling sites on Ascension Island:

> though far from probable, that the horse may have been born in the Isle of Wight. Even if we grant to animals a sense of the points of the compass, of which there is no evidence, how can we account, for instance, for the turtles which formerly congregated in multitudes, only at one season of the year, on the shores of the Isle of Ascension, finding their way to that speck of land in the midst of the great Atlantic Ocean?
>
> CHARLES DARWIN

Extract from “Perception in the Lower Animals”, C. Darwin, Nature 1873.

Classic example of a “homing problem”: other well-known examples include homing pigeons, lobsters, salmon, newts...

Green Turtle Navigation

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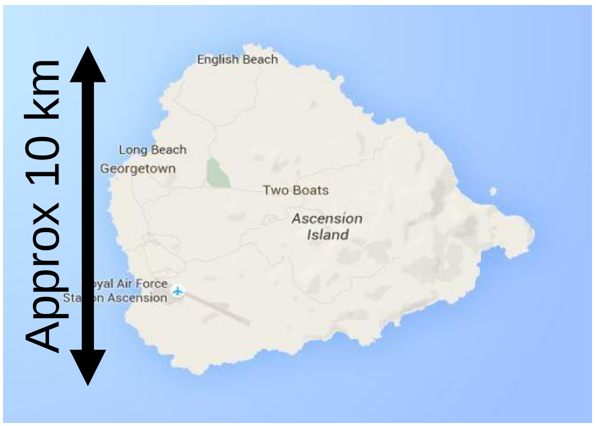
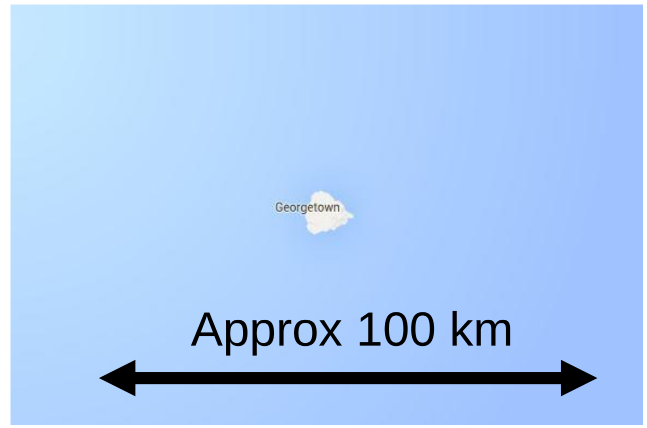
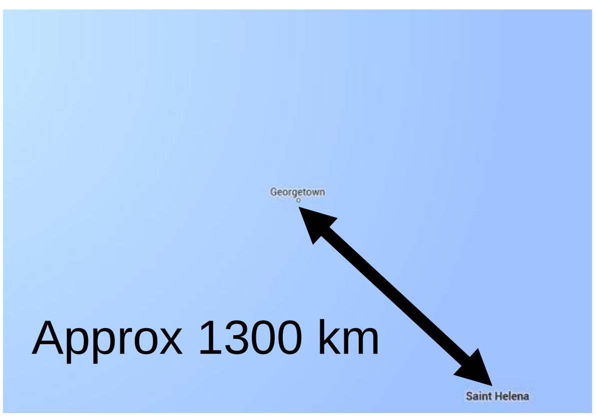


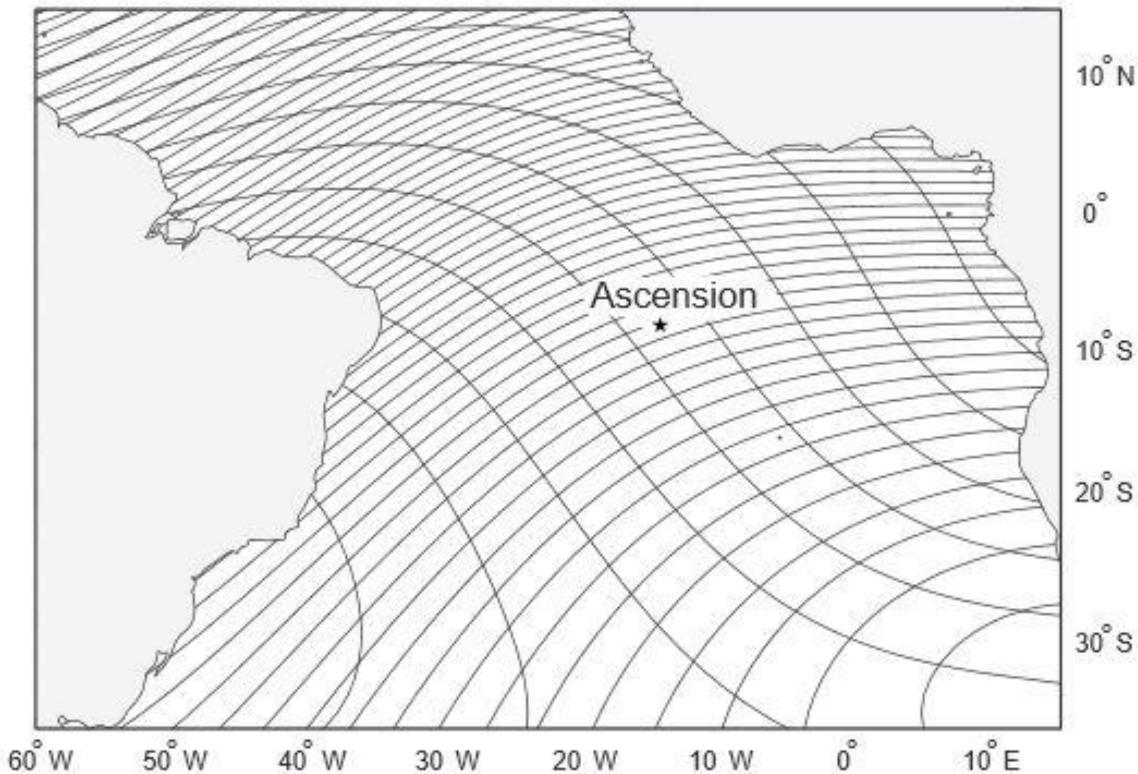




MACROSCOPIC SCALE PROBLEM.

Green Turtle Navigation: Theories

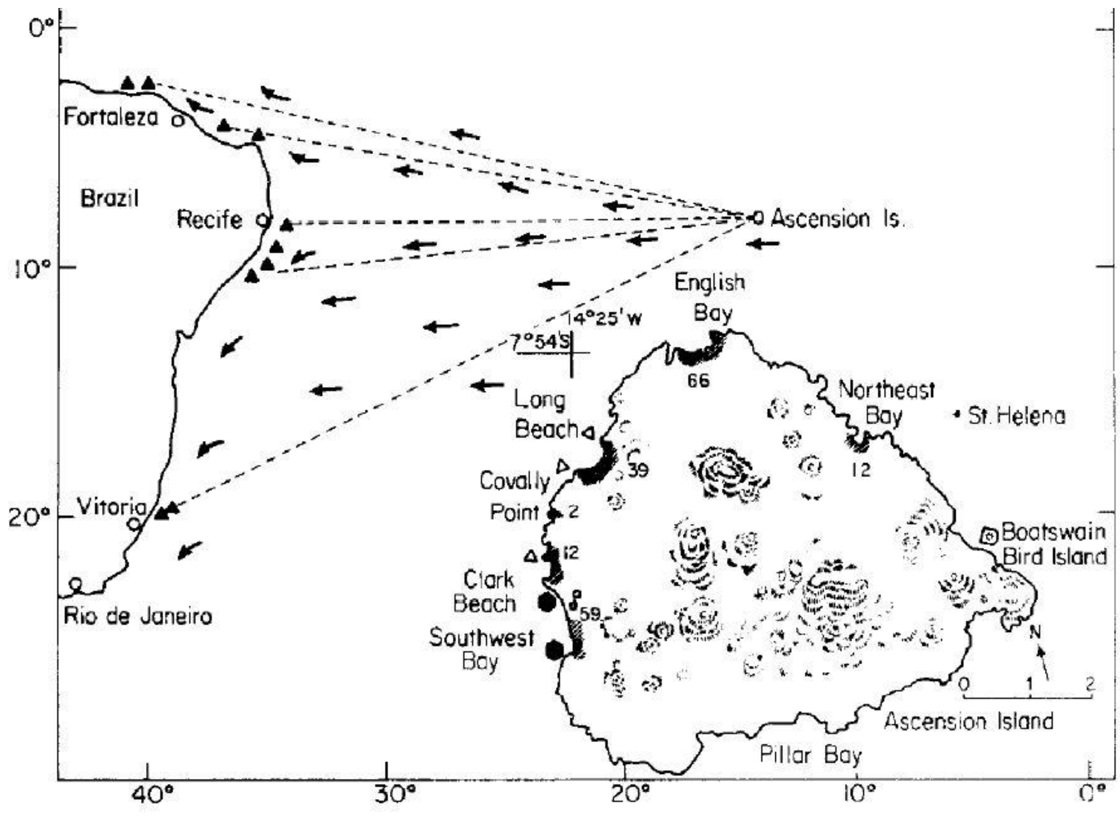
- Geomagnetic information?



Isoclinics and isodynamics in South Atantic (Lohmann 1999).

Green Turtle Navigation: Theories

- Geomagnetic information?
- Chemical cues borne by wind or ocean currents?



Green Turtle Navigation: Theories

- Geomagnetic information?
- Chemical cues borne by wind or ocean currents?
- Orientation with respect to the flow (rheotaxis)?
- Celestial information?
- Sight, sound (when sufficiently close)?
- But: how precise can any of these be?

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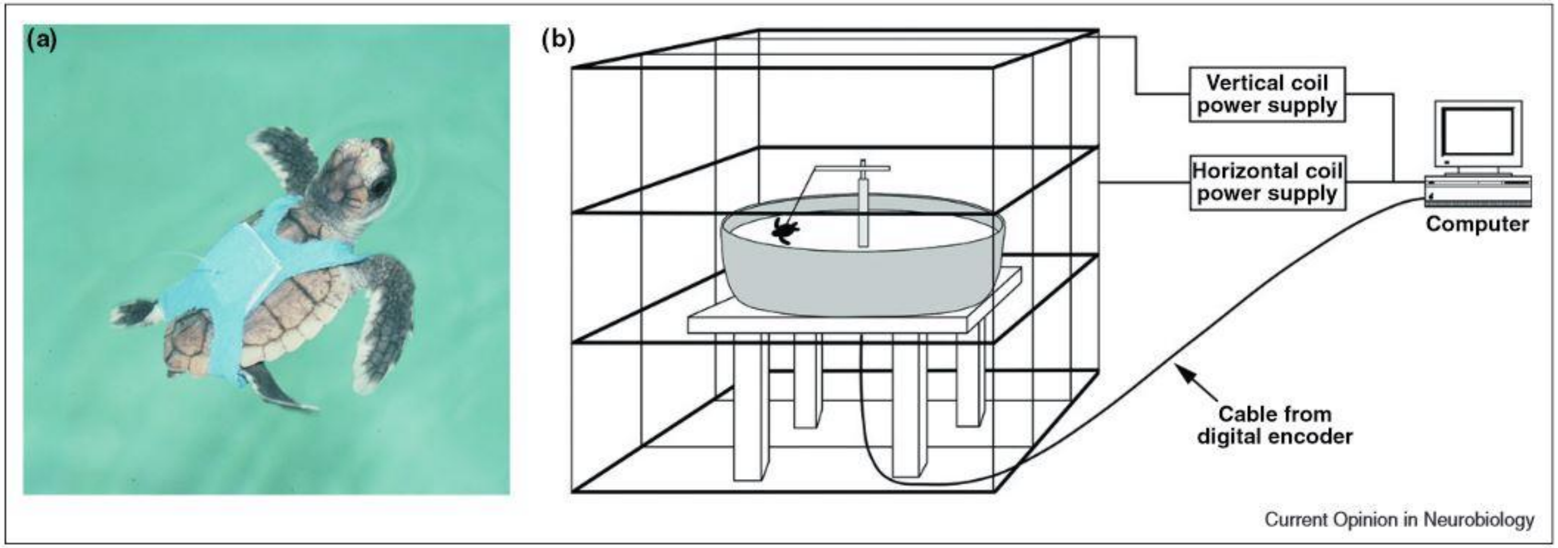
Green Turtle Navigation: Data

Controlled behavioural studies:

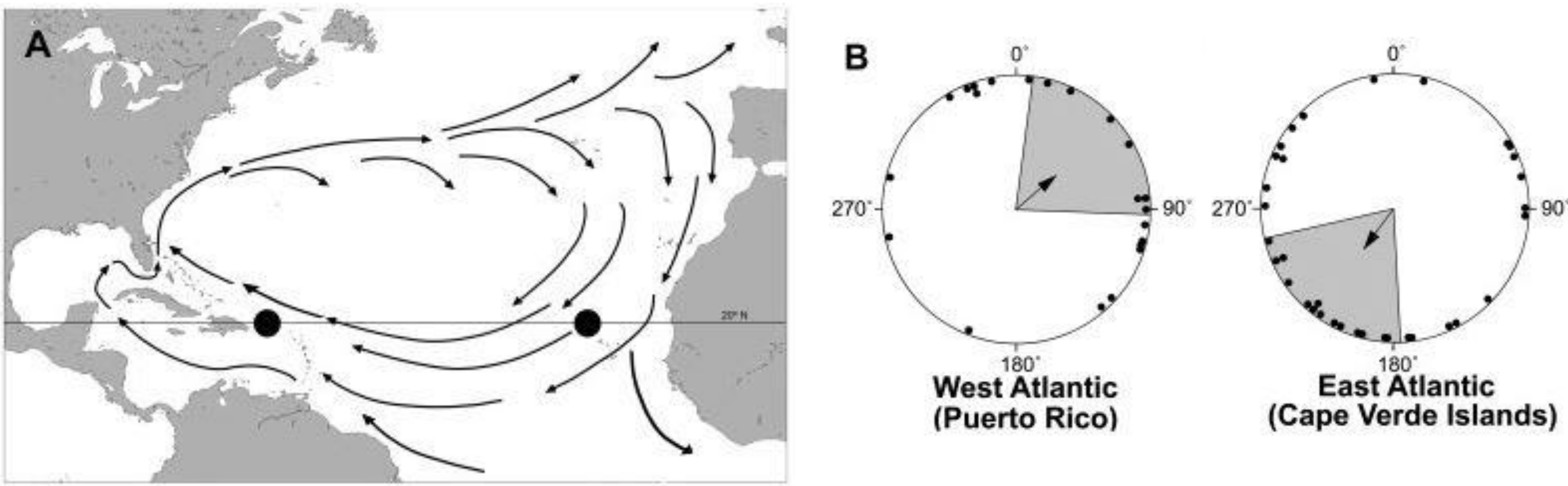


Experimental apparatus used to monitor orientation responses of hatchling loggerhead turtles to regional magnetic fields. **(A)** A hatchling loggerhead swimming in a cloth harness. **(B)** Turtles were tethered to a rotatable tracker arm attached to a digital encoder, which relayed the direction of swimming to a computer in a nearby building. The arena was enclosed by a magnetic coil system capable of replicating magnetic fields found at different locations along the turtles' migratory route. Modified from Lohmann and Lohmann [23].

Testing impact of geomagnetic field information on hatchling turtle navigation (Lohmann 2008)

Green Turtle Navigation: Data

Controlled behavioural studies:



Circular statistics analysis: can be used to generate turning distributions to assess strength of navigating information...

Of course, one should be careful about reading too much information into how such studies translate to "real-life"

Green Turtle Navigation: Data

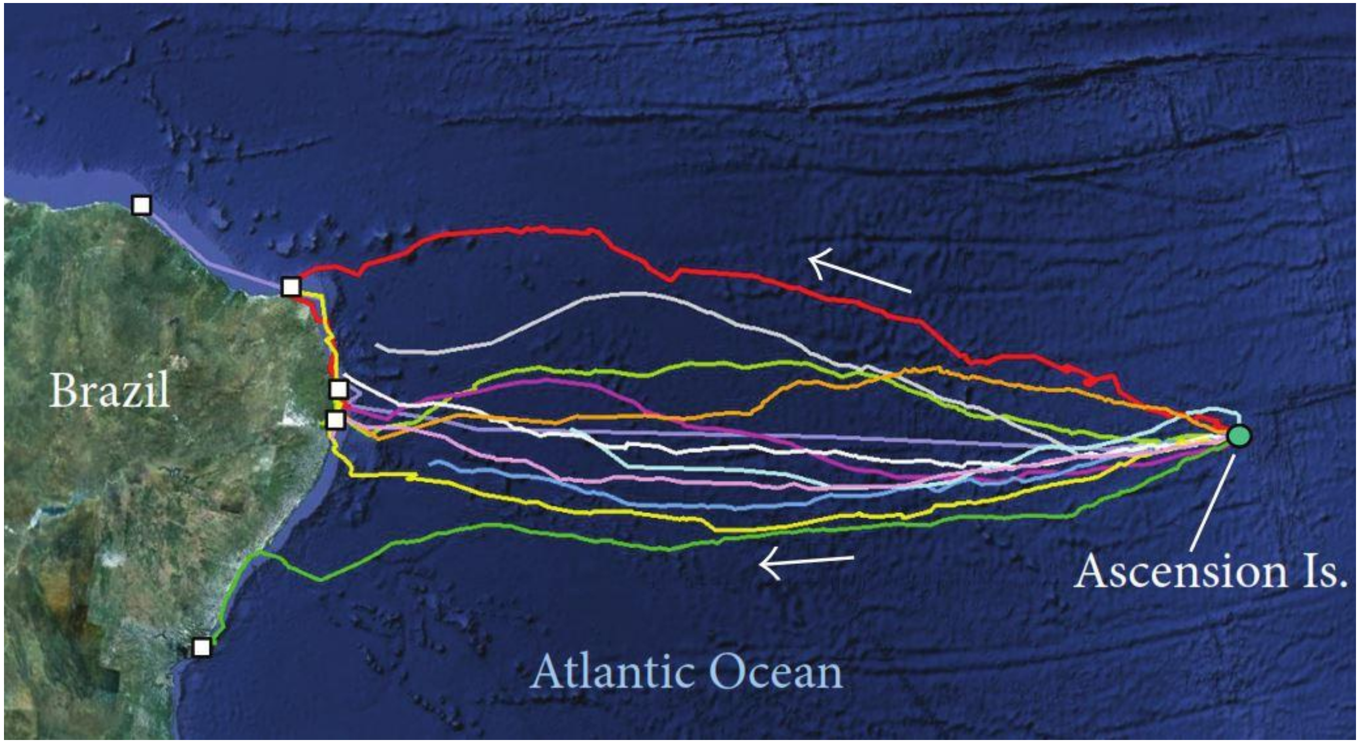
"Tag, release" experiments:

1950s: Helium balloons attached to nesting turtles.

1990s: Advent of GPS tracking

Green Turtle Navigation: Data

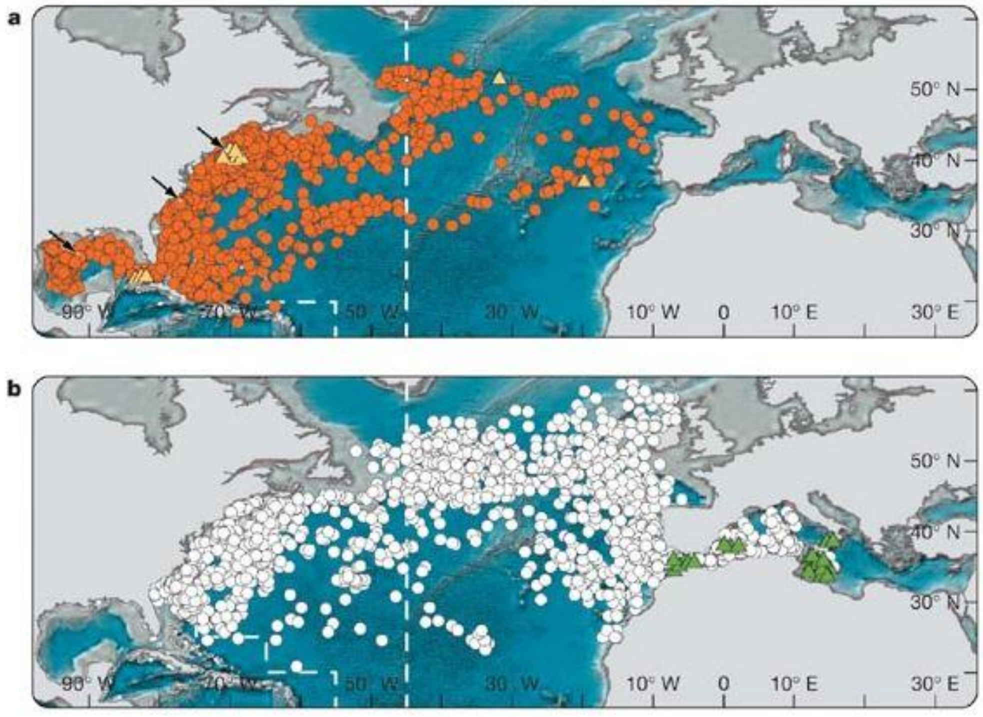
"Tag, release" experiments:



Individual-level data: generates mean speeds, turning rates, angles...

Green Turtle Navigation: Data

"Tag, release" experiments:



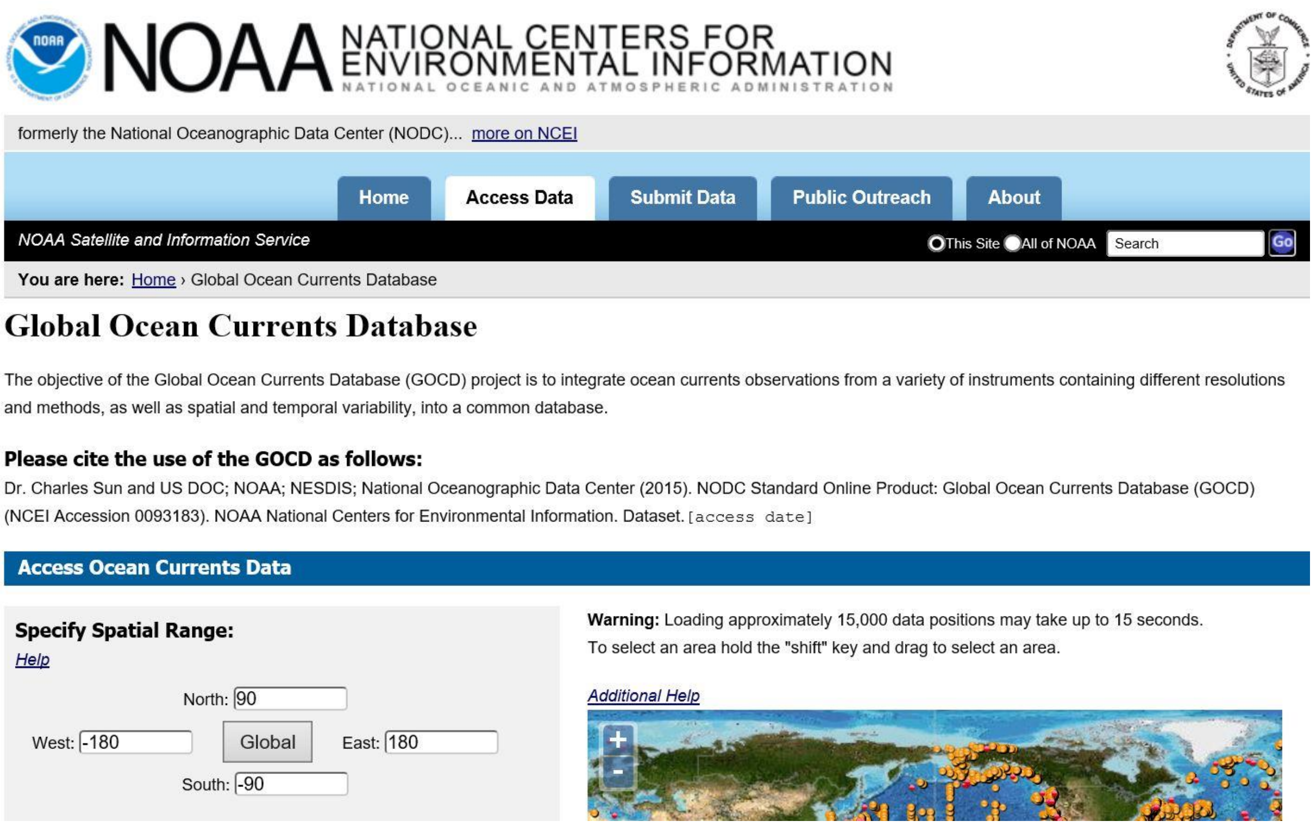
Block et al (2005), Nature. Electronic tagging of 800 tuna.

For commercially-important species, can even generate population level distributions. Not feasible for turtles….

Green Turtle Navigation: Data

Ocean currents: publicly available data available from:

1. Direct measurements (e.g. buoys, floats, ships, satellite,…)
2. Validated ocean circulation models...



Green Turtle Navigation: Data

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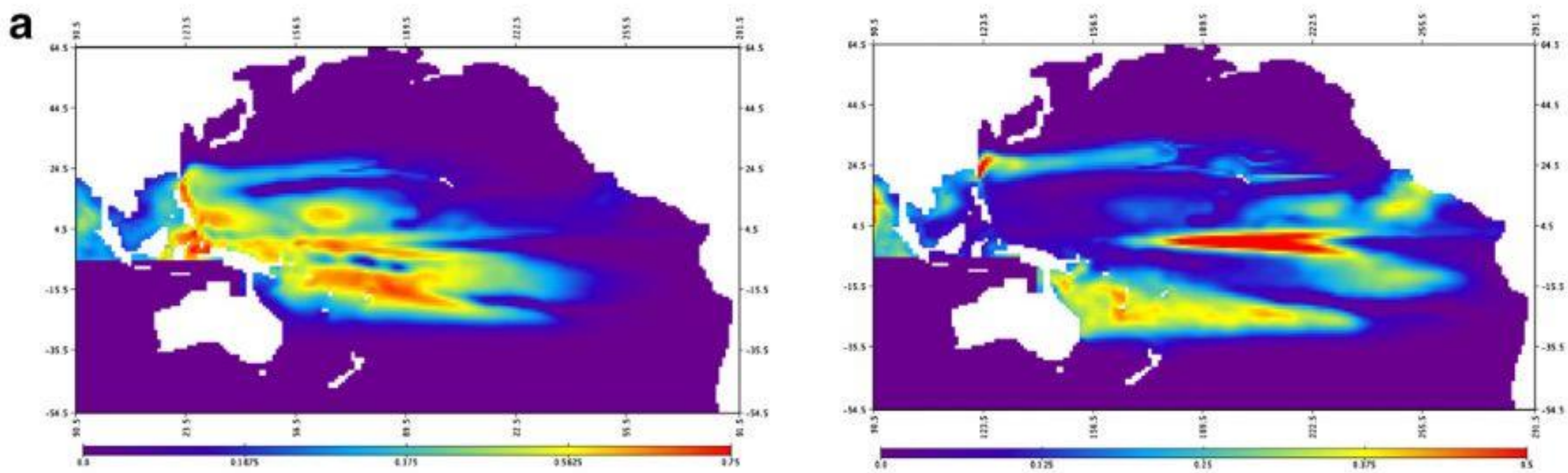
1. Direct measurements (e.g. buoys, floats, ships, satellite,…)
2. Validated ocean circulation models...

Oceanic flow and surface temperature data (ECCO2, NASA)

Modelling: fully continuous

Both currents and populations modelled as continuous variables.

Typically take the form of diffusion-advection-reaction systems



Ledohey et al (2008). SEAPODYM. Dynamic modelling of tuna population distributions.

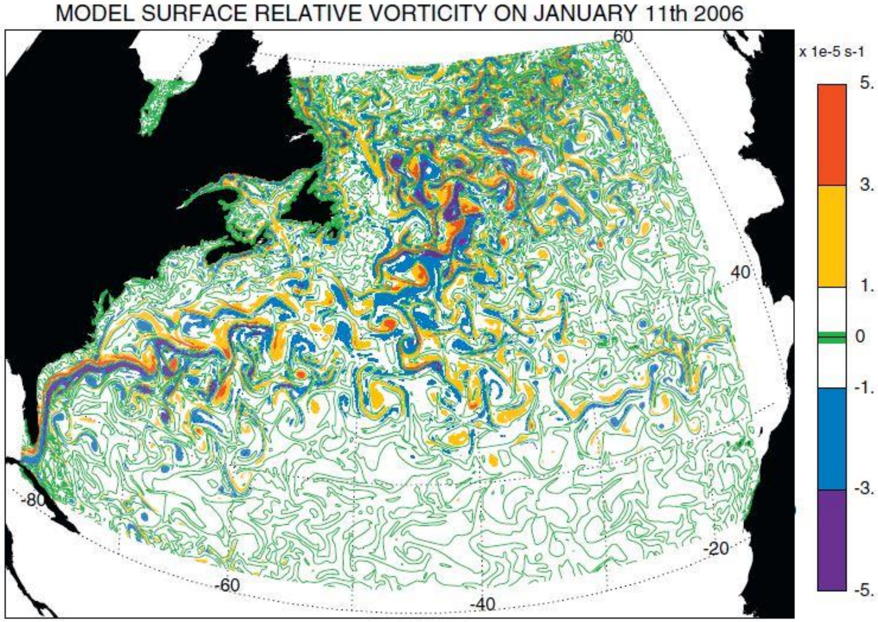
+'s: Numerically efficient, (somewhat) analytically tractable, directly generate population level detail.

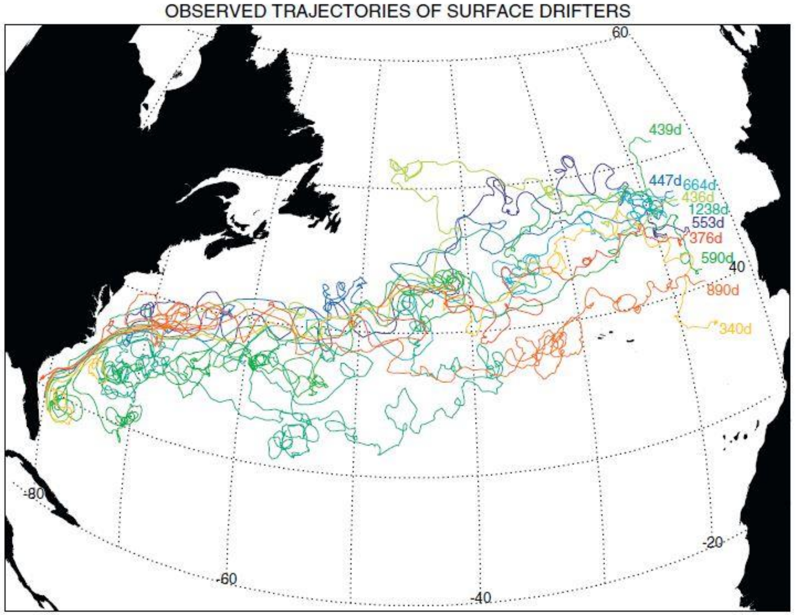
-'s: Hard to parameterise/validate against individual-level data

Modelling: IBM-continuous hybrid

Individuals immersed as (typically point) Lagrangian particles into a continuous flow. Individual movement could be:

1. Simple drifting (e.g. eggs, larvae,...)
2. Active swimming and navigating (fish, turtles…)





Blanke et al (2012): European eel larvae dispersal modelled as drifters

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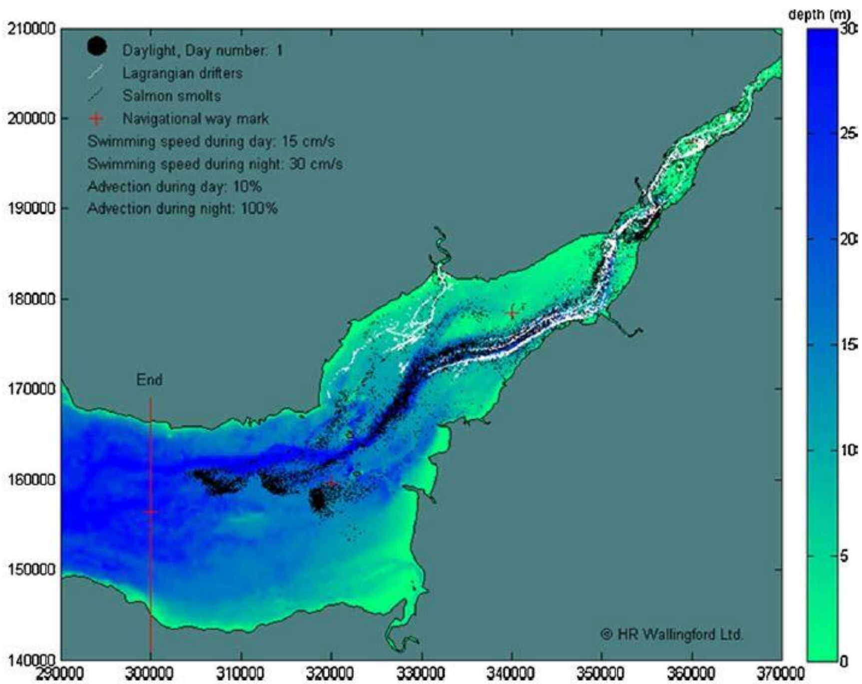
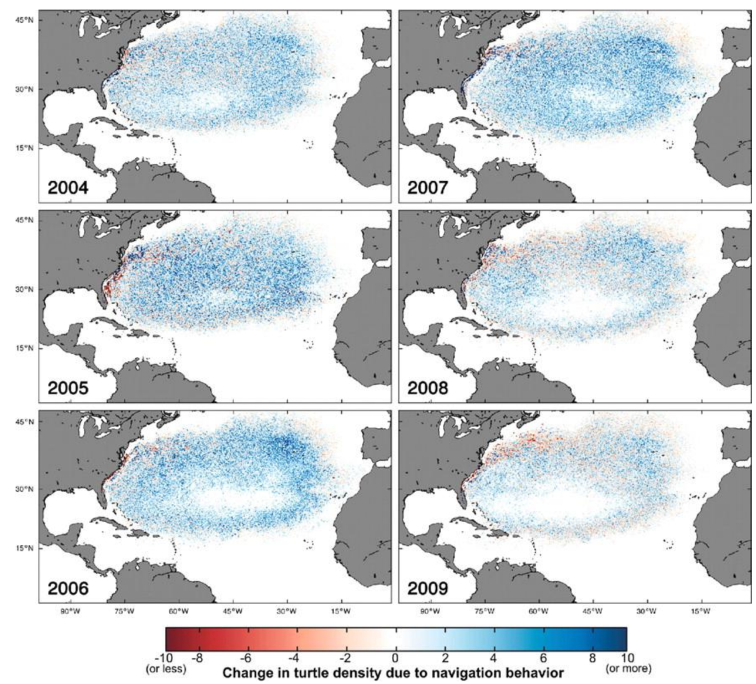


Willis (2011): Salmon in Severn estuary



Putman (2010): Turtle hatchling navigation

Modelling: IBM-continuous hybrid

Individuals immersed as (typically point) Lagrangian particles into a continuous flow. Individual movement could be:

1. Simple drifting (e.g. eggs, larvae,...)
2. Active swimming and navigating (fish, turtles…)

Widely adopted in both academic (animal orientation) and commercial sectors (fish stock forecasting).

+'s: Easy to parameterise against individual-level data; can tailor to incorporate specific individual movement rules; can obtain data at the level of both individuals and populations.

-'s: Computationally intensive (at the population level) and analytically intractable. Difficult to obtain non-parameter specific predictions.

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- A multiscale approach: from a hybrid IBM to a fully continuous model
- Application to homing.

Multiscale approach

Hybrid IBM to describe movement in a flow

↓

Reformulate as "mesoscopic" level transport model

↓

Scale to obtain macroscopic/ population-level model

+'s: Formulated, parametrised and validated against individual level data

+'s: A continuous-level model formulated at an individual movement scale

+'s: Analytically & numerically efficient for population-level /macroscopic understanding

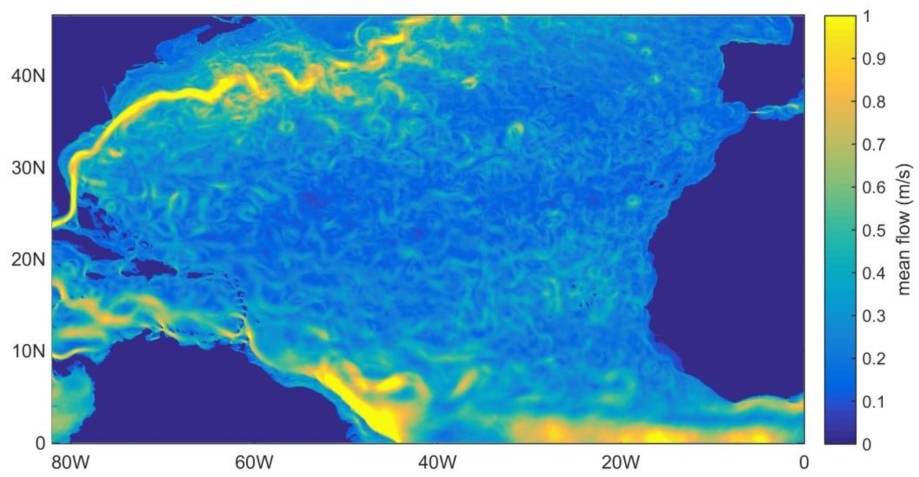
It's not all rosy! There are caveats: simplifying approaches must be made and hence its validity will depend on the problem…..

Multiscale approach: hybrid IBM

Individuals immersed as point-wise particles into an external flow.

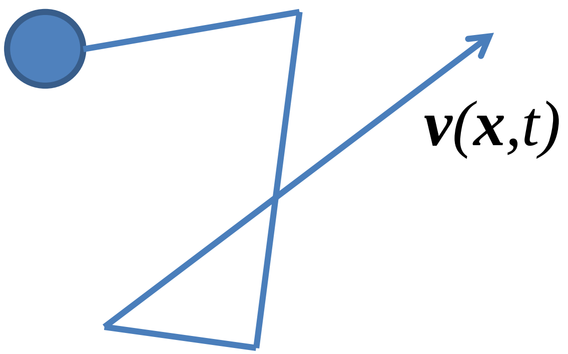
Particles do not interact and have negligible impact on the flow.

Individual movement from "passive" (external flow) and "active" (swimming /flying) motion:



Passive velocity field $\boldsymbol{u}(\boldsymbol{x},t)$

Velocity-jump model



Active velocity $\boldsymbol{v}(\boldsymbol{x},t)$

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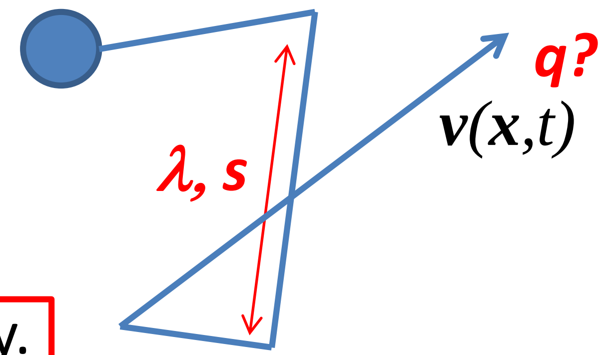

<u>Key defining parameters/functions:</u>

λ = mean run time;

s = mean speed;

q = probability distribution for new velocity.

→ Can be biased to incorporate navigating information

Multiscale approach: hybrid IBM

Individuals immersed as point-wise particles into an external flow.

Particles do not interact and have negligible impact on the flow.

Individual movement from "passive" (external flow) and "active" (swimming /flying) motion:

Let $\boldsymbol{x}_i(t), \boldsymbol{v}_i(t)$ be the position and velocity of an individual i at time t.

Let $\boldsymbol{u}(t, \boldsymbol{x})$ be the external flow at time t, position $\boldsymbol{x}$.

Then for a sufficiently short time step, Δt,

$$\boldsymbol{x}_i(t+\Delta t) = \boldsymbol{x}_i(t) + \Delta t(\boldsymbol{v}_i(t) + \boldsymbol{u}(t, \boldsymbol{x}_i(t)))$$

$$\boldsymbol{v}_i(t+\Delta t) = \begin{cases} q\left(\boldsymbol{v} \middle| t+\Delta t, \boldsymbol{x}_i(t+\Delta t)\right) & prob.\, \lambda \Delta t \\ \boldsymbol{v}_i(t) & otherwise \end{cases}$$

Multiscale approach: mesoscopic

Rewrite* the IBM as a mesoscopic-level continuous transport model:

$$p_t(t,\mathbf{x},\mathbf{v}) + \overbrace{\nabla\cdot\mathbf{v}p(t,\mathbf{x},\mathbf{v})}^{\text{active movement}} + \overbrace{\nabla\cdot\mathbf{u}p(t,\mathbf{x},\mathbf{v})}^{\text{passive movement}} = \overbrace{\lambda(m(t,\mathbf{x})q(\mathbf{v}\,|\,t,\mathbf{x}) - p(t,\mathbf{x},\mathbf{v}))}^{\text{turning}}$$

$p(t, x, v) =$ particle density with time t, position x and velocity v

$m(t, x) =$ macroscopic (or observable) density

$$m(t,\mathbf{x}) = \int_V p(t,\mathbf{x},\mathbf{v})d\mathbf{v}$$

* Note: strictly speaking, this is valid only if interactions are “negligible”

Multiscale approach: macroscopic

For macroscopic problems, scaling (see Hillen/P. 2014) used to generate a macroscopic model. Using *moment closure**,

$$m(t,\mathbf{x})_t + \nabla \cdot ((\overset{①}{\mathbf{a}(t,\mathbf{x})} + \overset{②}{\mathbf{u}(t,\mathbf{x})}) m(t,\mathbf{x})) = \nabla\nabla\,(\underset{③}{\mathbb{D}(t,\mathbf{x})} m(t,\mathbf{x}))$$

① Active advection due to swimming/flying

② Passive advection due to flow

③ Anisotropic diffusion (uncertainty in active navigation)

*original methods developed in a physical context and are based on notions of mass, momentum, energy etc. Do these a transfer to biology context? Requires some moment closure condition: here we use a fast flux approximation which states that at the space/time scales of observation, individuals almost instantaneously respond to their environment.

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$$m(t,\mathbf{x})_t + \nabla \cdot ((\mathbf{a}(t,\mathbf{x}) + \mathbf{u}(t,\mathbf{x}))m(t,\mathbf{x})) = \nabla\nabla(\mathbb{D}(t,\mathbf{x})m(t,\mathbf{x}))$$

$$\nabla\nabla\mathbf{A}(\mathbf{x}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 \mathbf{A}}{\partial x_i x_j}$$

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$$\mathbf{a}(t,\mathbf{x}) = \int_V \mathbf{v}q(\mathbf{v}\,|\,t,\mathbf{x})d\mathbf{v},$$

i.e. active advection is the expectation of the turning distribution!

$$\mathbb{D}(t,\mathbf{x}) = \frac{1}{\lambda}\int_V (\mathbf{v}-\mathbf{a}(t,\mathbf{x}))(\mathbf{v}-\mathbf{a}(t,\mathbf{x}))^T q(\mathbf{v}\,|\,t,\mathbf{x})d\mathbf{v},$$

i.e. anisotropic diffusion is proportional to the covariance matrix of the turning distribution!

Multiscale approach: macroscopic

IBM to describe movement in a (continuously) flowing field

↓

Reformulate as "mesoscopic" level transport model

↓

Scale to obtain macroscopic/ population-level model

Inputs: Key parameters/terms are mean speed, turning rate and turning distribution

Exactly the same inputs as the IBM

Macroscopic terms/parameters depend on statistical properties of the IBM inputs

Movement classes

Simplification: assume individuals move with the same mean speed, *s*. Therefore, at each turn, only a new active direction (or heading) is chosen and we specify a suitable directional distribution, $\tilde{q}$, over all possible directions.

$$
\begin{aligned}
\mathbf{a}(t,\mathbf{x}) &= \int_V \mathbf{v} q(\mathbf{v} \,|\, t,\mathbf{x}) d\mathbf{v}\,, \\
&= s\int_{\mathbb{S}^{n-1}} \mathbf{n}\tilde{q}(\mathbf{n} \,|\, t,\mathbf{x}) d\mathbf{n}\,; \\
\mathbb{D}(t,\mathbf{x}) &= \frac{1}{\lambda}\int_V (\mathbf{v}-\mathbf{a}(t,\mathbf{x}))(\mathbf{v}-\mathbf{a}(t,\mathbf{x}))^T q(\mathbf{v} \,|\, t,\mathbf{x}) d\mathbf{v}\,, \\
&= \frac{1}{\lambda}\int_{\mathbb{S}^{n-1}} (s\mathbf{n}-\mathbf{a}(t,\mathbf{x}))(s\mathbf{n}-\mathbf{a}(t,\mathbf{x}))^T \tilde{q}(\mathbf{n} \,|\, t,\mathbf{x}) d\mathbf{n}\,.
\end{aligned}
$$

Movement classes

- **"Drifters":** organisms that simply drift with the external flow, e.g. eggs, larvae and small organisms.
- Hence, no active speed (s = 0) and we obtain:

$$m(t, \mathbf{x})_t + \nabla \cdot \mathbf{u}(t, \mathbf{x}) m(t, \mathbf{x}) = 0$$

- As to be expected, a classic drift-equation....

Movement classes

- **"Random movers":** organisms move actively but (at the observed level) in an essentially random manner.
- Choose uniform probability distributions: defining (2D) polar angle α or (3D) azimuth/polar angles (α,β), set:

$$(2\text{D})\ \tilde{q}_c(\alpha) = 1/2\pi\,; \quad (3\text{D})\ \tilde{q}_s(\alpha,\beta) = 1/4\pi\,.$$

- Substitution and integrating gives:

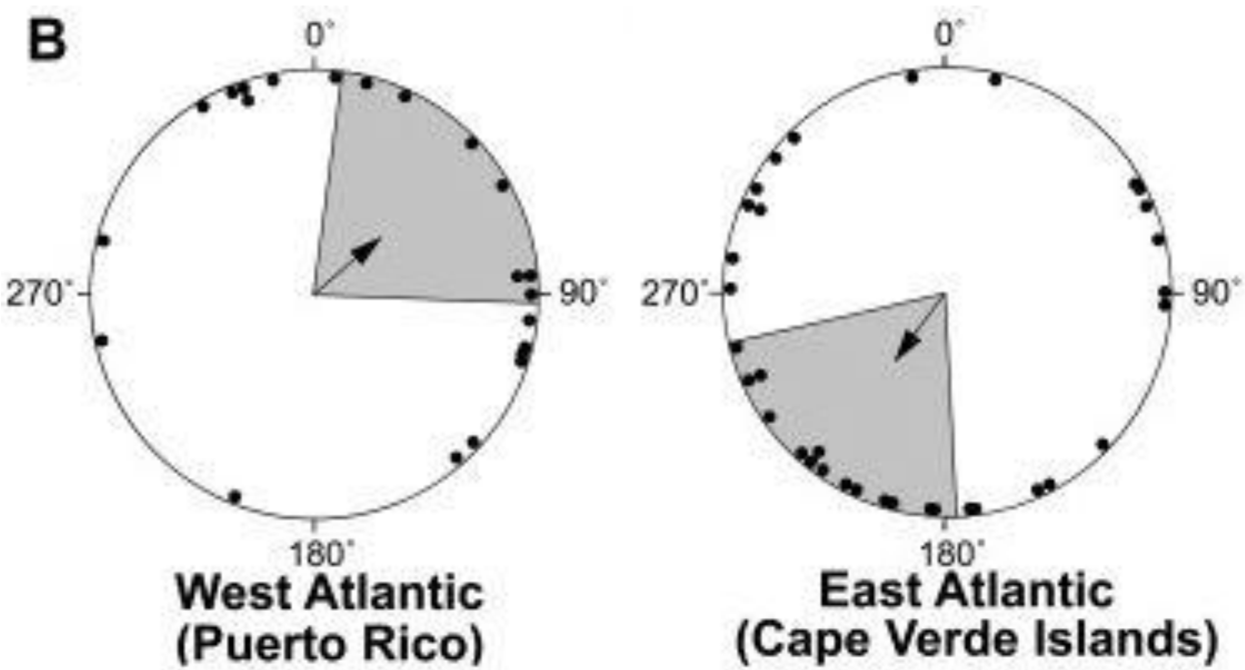
$$\mathbf{a}(t,\mathbf{x}) = \mathbf{0}\,, \quad \text{and} \quad \mathbb{D}(t,\mathbf{x}) = \frac{s^2}{n\lambda}\mathbb{I}_n$$

- Hence, a drift/isotropic diffusion equation:

$$m(t,\mathbf{x})_t + \nabla\cdot\mathbf{u}(t,\mathbf{x})m(\mathbf{x},t) = d\nabla^2 m(t,\mathbf{x}) \quad d = s^2/(n\lambda)$$

Movement classes

- **"Navigators":** organisms that actively move in response to environmental navigating cues.
- Example data: hatchling turtle data sets...



Movement classes

- **"Navigators":** organisms that actively move in response to environmental navigating cues.
- Example data: hatchling turtle data sets...
- "Standard choice" (for 2D) is the von Mises distribution: a circular analogue of the normal distribution:

$$\tilde{q}(\alpha \mid k, A) = \frac{1}{2\pi I_0(k)} e^{k\cos(\alpha - A)}$$

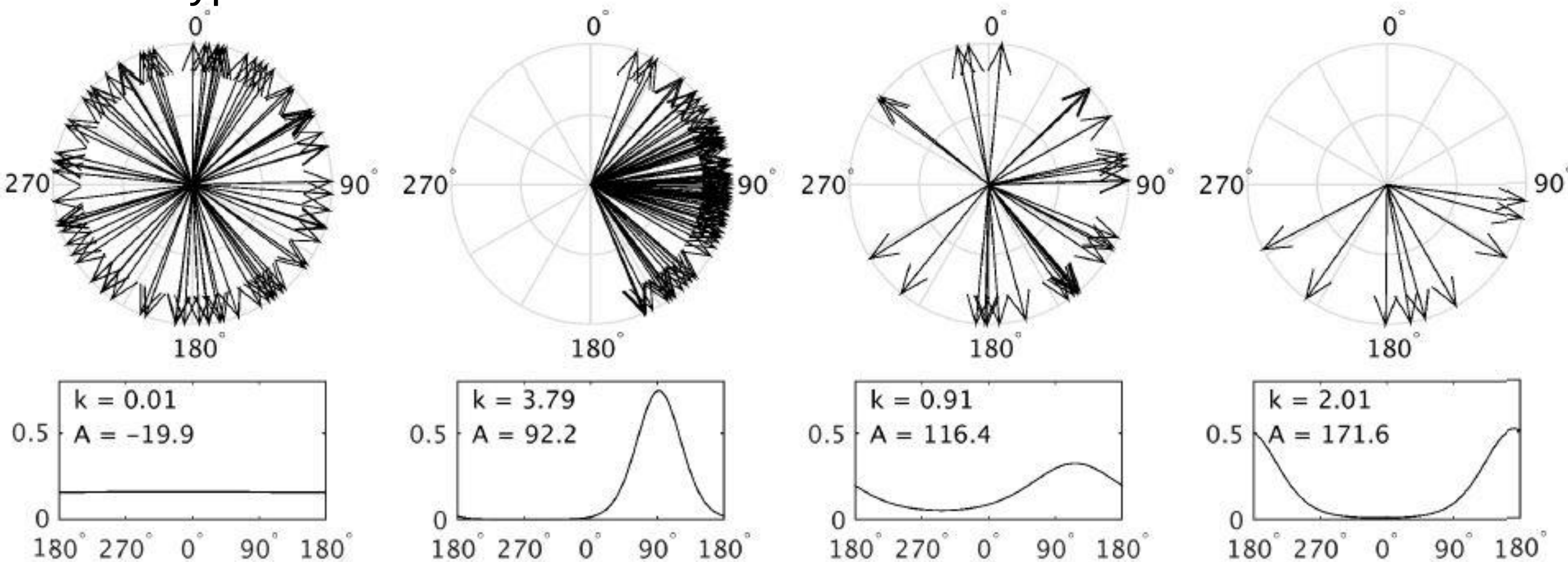
- *k* is a concentration parameter measuring the navigational strength, while *A* defines the desired directional choice.
- $I_j(k)$ denotes the modified Bessel function of order *j*.

Movement classes

- **“Navigators”:** organisms that actively move in response to environmental navigating cues

$$\tilde{q}(\alpha \mid k, A) = \frac{1}{2\pi I_0(k)} e^{k\cos(\alpha - A)}$$

Hypothetical data sets   Genuine data sets



Analysis of turtle magnetic orientation datasets: k values 0.5 - 2

Movement classes

- **"Navigators":** organisms that actively move in response to environmental navigating cues.
- "Fun" integral calculations....

$$\mathbf{a}(t,\mathbf{x}) = s\int_{\mathbb{S}^{n-1}} \mathbf{n}\tilde{q}(\mathbf{n}\,|\,t,\mathbf{x})d\mathbf{n}$$

$$\mathbb{D}(t,\mathbf{x}) = \frac{1}{\lambda}\int_{\mathbb{S}^{n-1}} (s\mathbf{n}-\mathbf{a}(t,\mathbf{x}))(s\mathbf{n}-\mathbf{a}(t,\mathbf{x}))^T\tilde{q}(\mathbf{n}\,|\,t,\mathbf{x})d\mathbf{n}$$

Movement classes

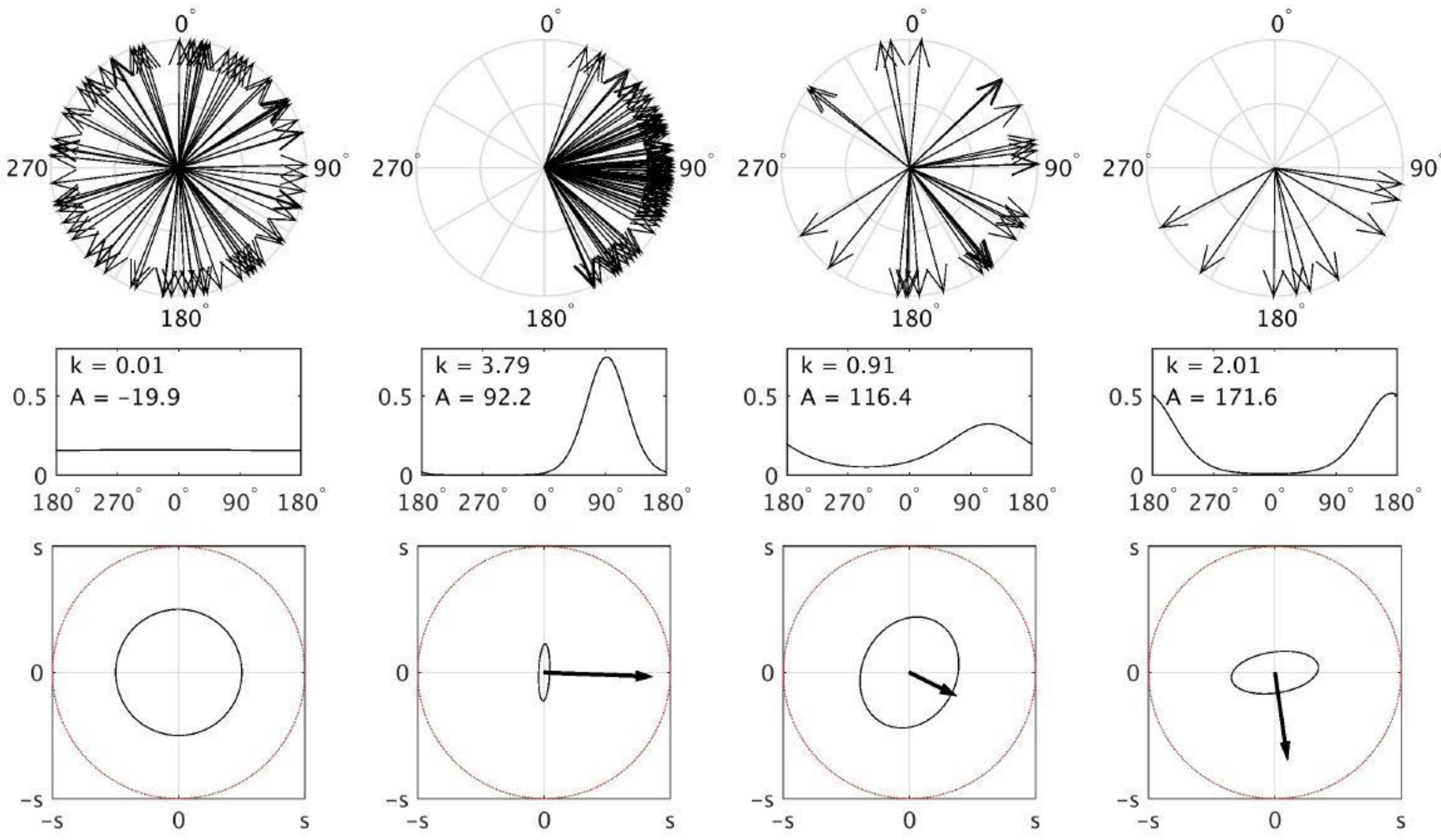
- **"Navigators":** organisms that actively move in response to environmental navigating cues.
- "Fun" integral calculations....

$$
\begin{aligned}
\mathbf{a}(k,A) &= s\frac{I_1(k)}{I_0(k)}(\cos A, \sin A)\,, \\
\mathbb{D}(k,A) &= \frac{s^2}{2\lambda}\left(1-\frac{I_2(k)}{I_0(k)}\right)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&\quad + \frac{s^2}{\lambda}\left(\frac{I_2(k)}{I_0(k)}-\frac{I_1(k)^2}{I_0(k)^2}\right)\begin{pmatrix} \cos^2 A & \cos A \sin A \\ \cos A \sin A & \sin^2 A \end{pmatrix}
\end{aligned}
$$

- Advection in direction of dominant angle (obviously).
- Diffusion is anisotropic, depending on *A*.

Movement classes

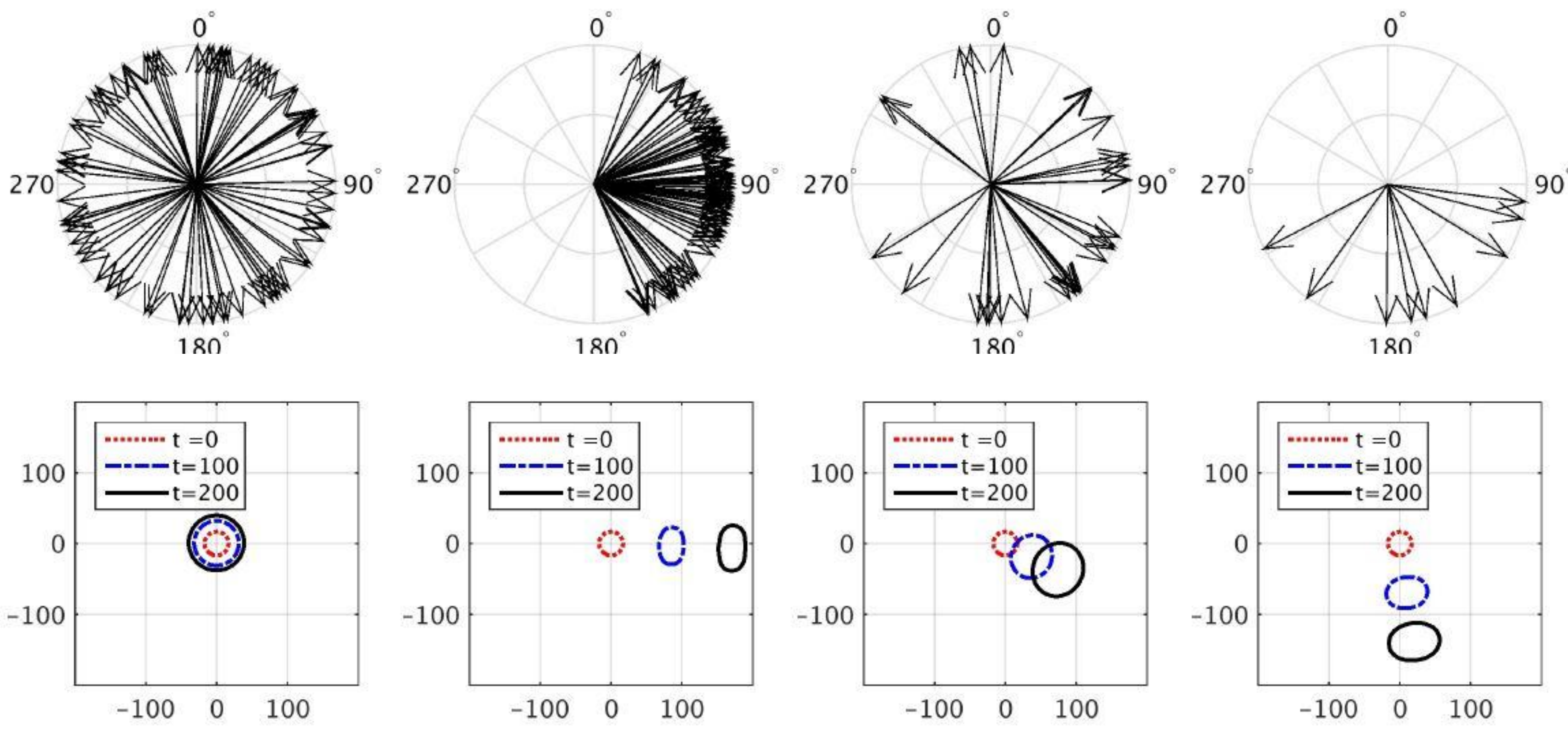
- **“Navigators”:** organisms that actively move in response to environmental navigating cues.



Movement classes

- **“Navigators”:** organisms that actively move in response to environmental navigating cues.
- Substitute into macroscopic model and solve...

$$m(t,\mathbf{x})_t + \nabla\cdot((\mathbf{a}(t,\mathbf{x})+\mathbf{u}(t,\mathbf{x}))m(t,\mathbf{x})) = \nabla\nabla\left(\mathbb{D}(t,\mathbf{x})m(t,\mathbf{x})\right)$$



Movement classes

- **"Navigators":** organisms that actively move in response to environmental navigating cues.
- Can also be done for 3D data sets. Uses the spherical von-Mises/Fisher distribution:

$$\tilde{q}(\alpha, \beta \,|\, k, A, B) = \frac{k}{4\pi \sinh k} e^{k(\cos\beta \cos B + \sin\beta \sin B \cos(A-\alpha))}$$

- Even more fun calculations give...

$$\begin{aligned} \mathbf{a}(k, A, B) &= s\left(\coth k - \tfrac{1}{k}\right)\mathbf{w}\,, \\ \mathbb{D}(k, A, B) &= \tfrac{s^2}{\lambda}\left(\left(\tfrac{\coth k}{k} - \tfrac{1}{k^2}\right)\mathbb{I}_3 + \left(1 - \tfrac{\coth k}{k} + \tfrac{2}{k^2} - \coth^2 k\right)\mathbf{w}\mathbf{w}^T\right) \end{aligned}$$

- Important for problems with both vertical and lateral movement….

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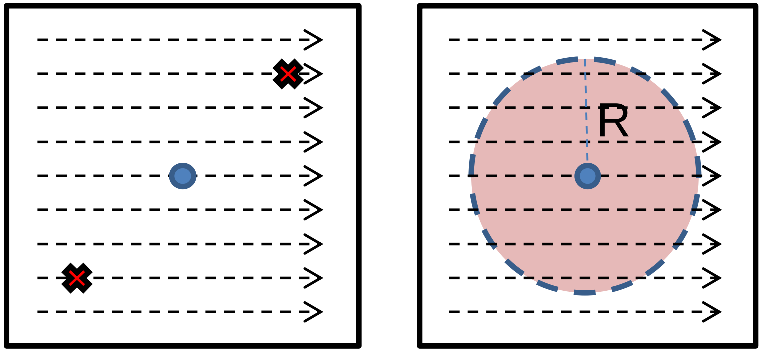
- A multiscale approach: from a hybrid IBM to a fully continuous model
- **Application to homing.**

Goal Navigation

- **Problem**: navigation to a specific goal under external flows.
  - Marine turtle navigation to island beaches...
  - Moths flying to a pheromone source...
  - Salmon returning from oceans to freshwater rivers...
- Flows can easily exceed an individual's swimming/flying speed.
- Individuals must correct or compensate for the flow in order to home.
  - Correction requires periodic reassessment of active heading, but not actual detection of the flow.
  - Compensation requires actual detection of the external flow.
- Here we assume correction only (believed to be the case for turtles).
- *How strong does the orienteering capacity need to be?*

Goal Navigation: Idealised

- Constant and uniform flow $\mathbf{u} = (u_x, 0)$ from west to east.
- Assume circular goal of radius r centred on the origin.
- I.C.s: releases NE/SW from the goal or uniformly distributed.

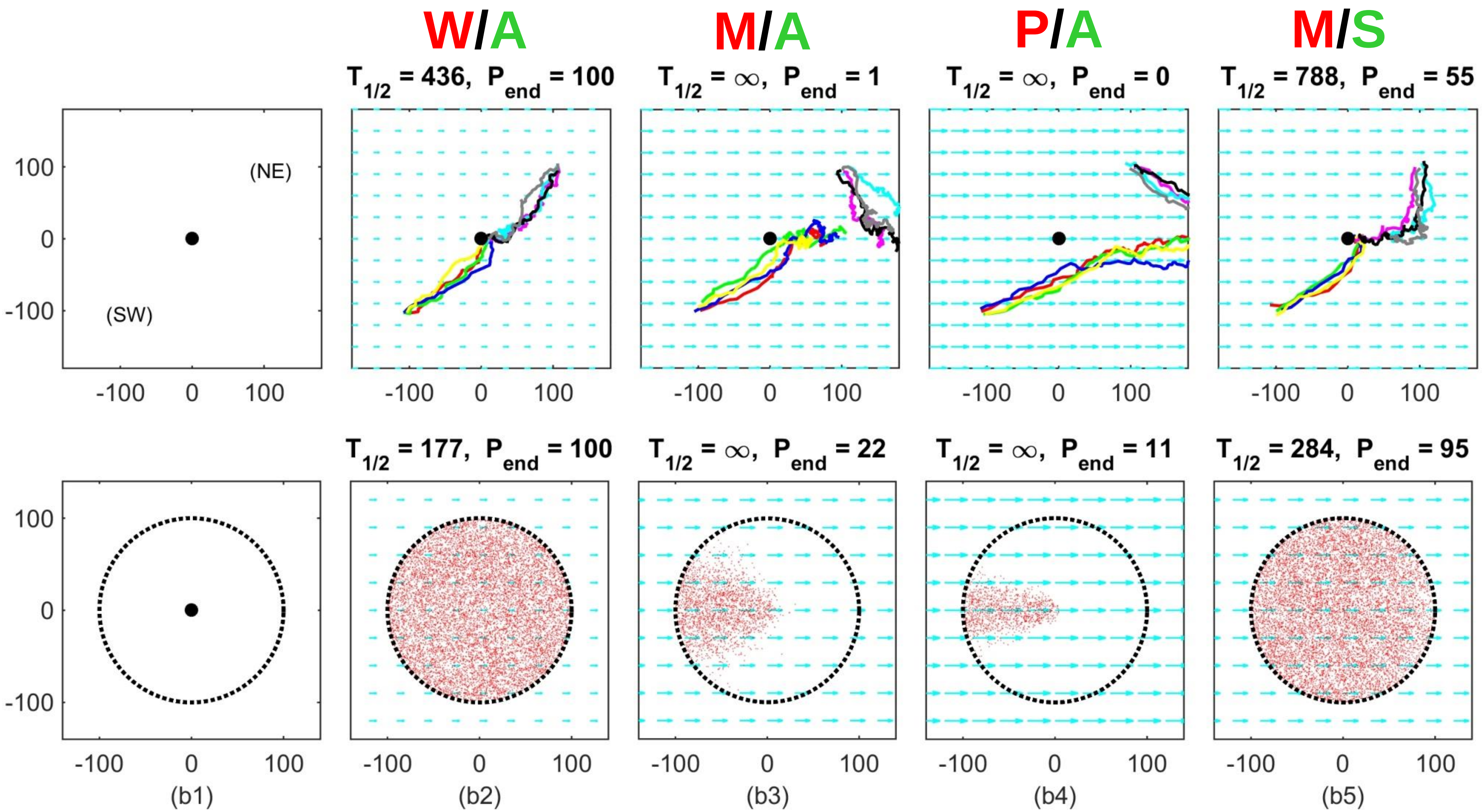


- Navigating: use VM dist. with dominant direction pointing to the origin and constant navigating strength k.
- Non-dimensional form: set *s* = λ = *1, r* = *5, R* = *100*.
- Key parameters: flow u_x and navigating strength k.
- Success Measure: % that reaches target by t = T (=1000).

Goal Navigation: Idealised

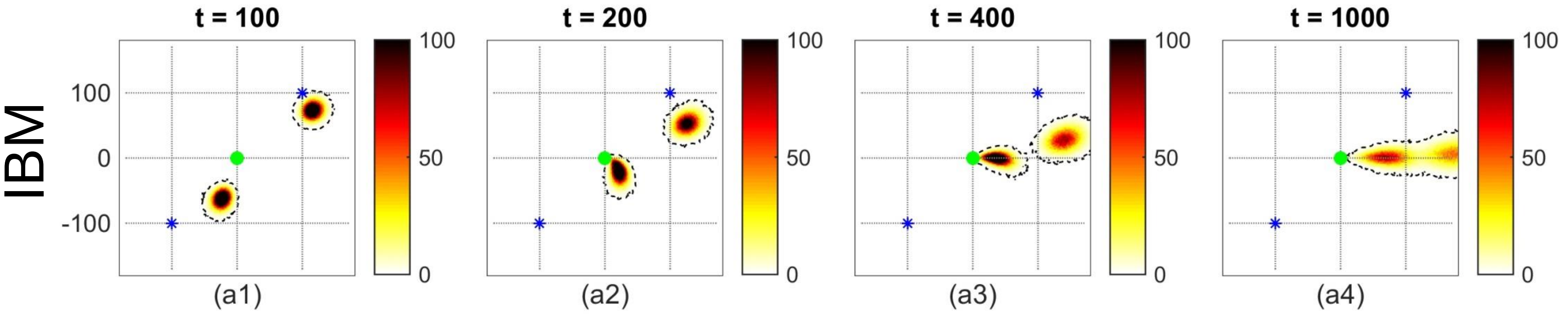
Flows: Weak (25% of s); Moderate (50%); Powerful (75%).

Navigation: Average (k = 1), Strong (k = 1.5).



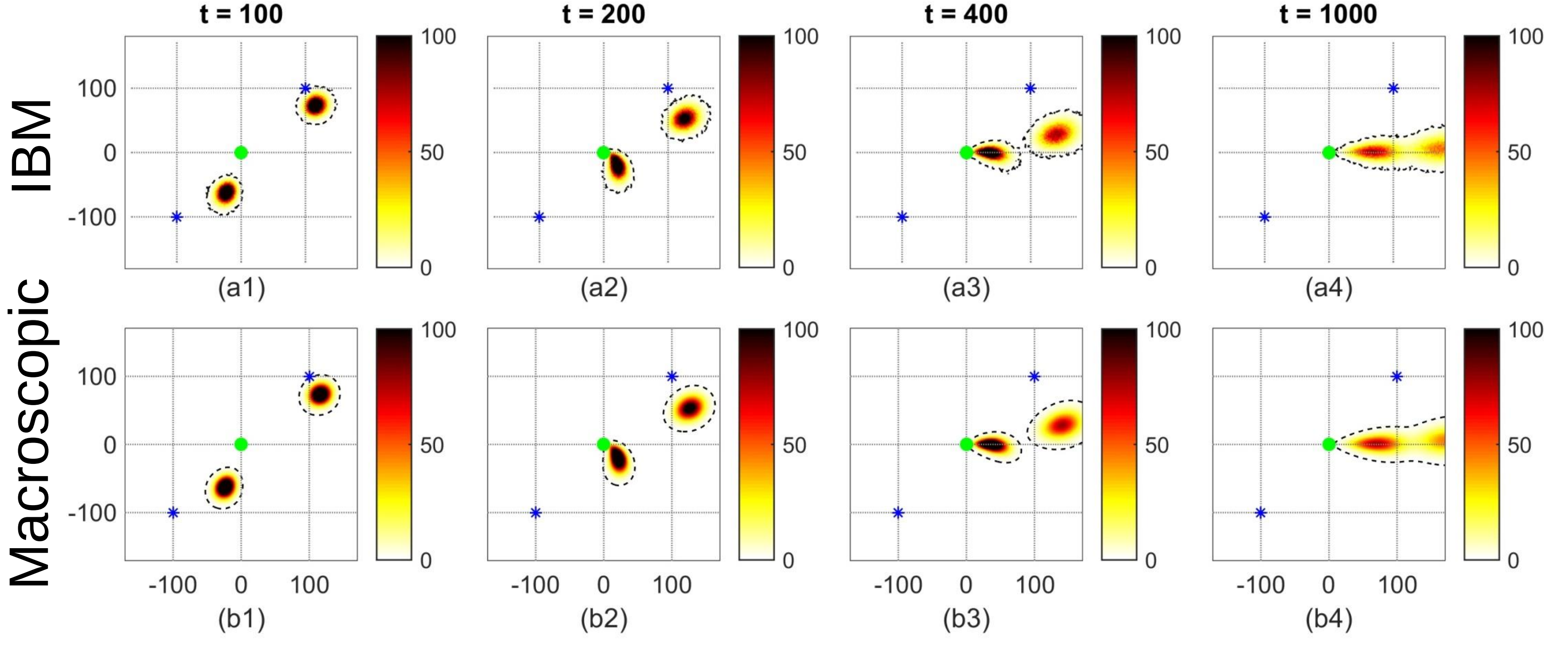
Goal Navigation: Idealised

Does the macroscopic model provide an acceptable representation of IBM population level statistics?



Goal Navigation: Idealised

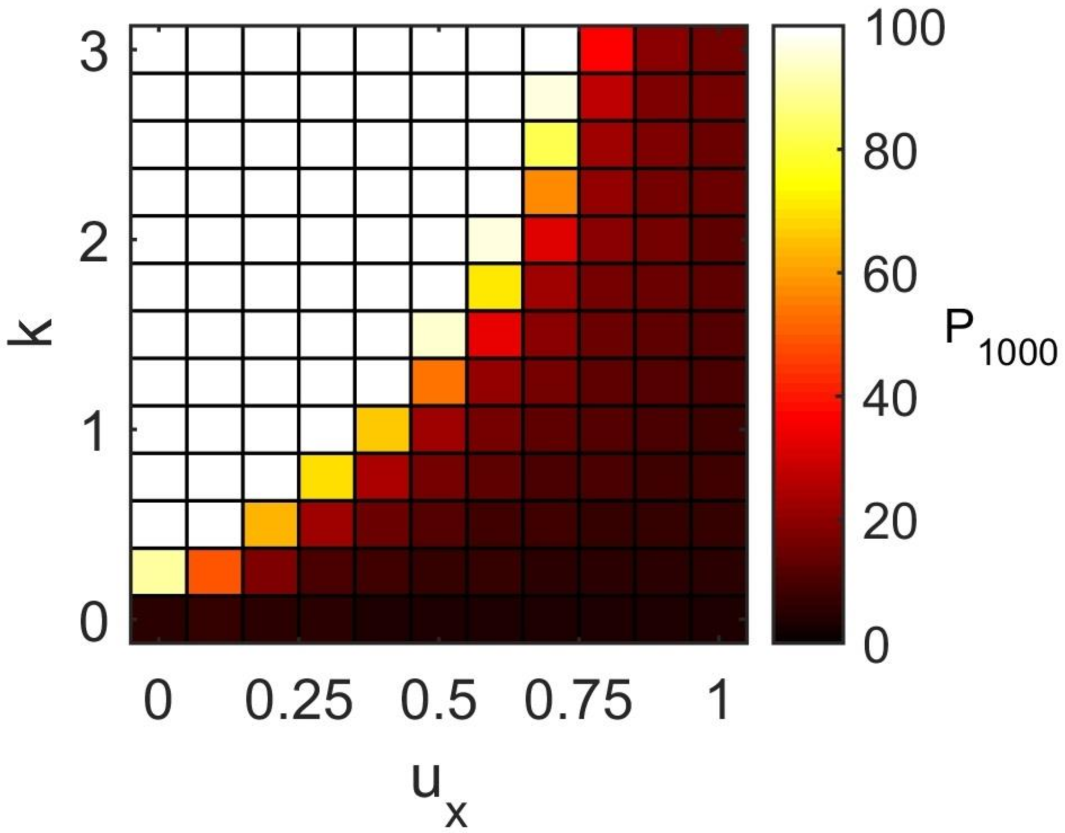
Does the macroscopic model provide an acceptable representation of IBM population level statistics?



Hence, can exploit the superior analytical efficiency of the macroscopic model.

Goal Navigation: Idealised

Parameter sweep through navigational strength/flow speed space to assess population homing success:



Seemingly sharp threshold between k and u_x: quantifiable?

Goal Navigation: Idealised

Use method of characteristics: gives a continuous dynamical system for the path, **x**(t) = (x(t),y(t)), of an "average" individual.

For the macroscopic model we have:

$$\frac{d\mathbf{x}}{dt} = \mathbf{u}(t,\mathbf{x}) + \mathbf{a}(t,\mathbf{x}) - \nabla \cdot \mathbb{D}(t,\mathbf{x})$$

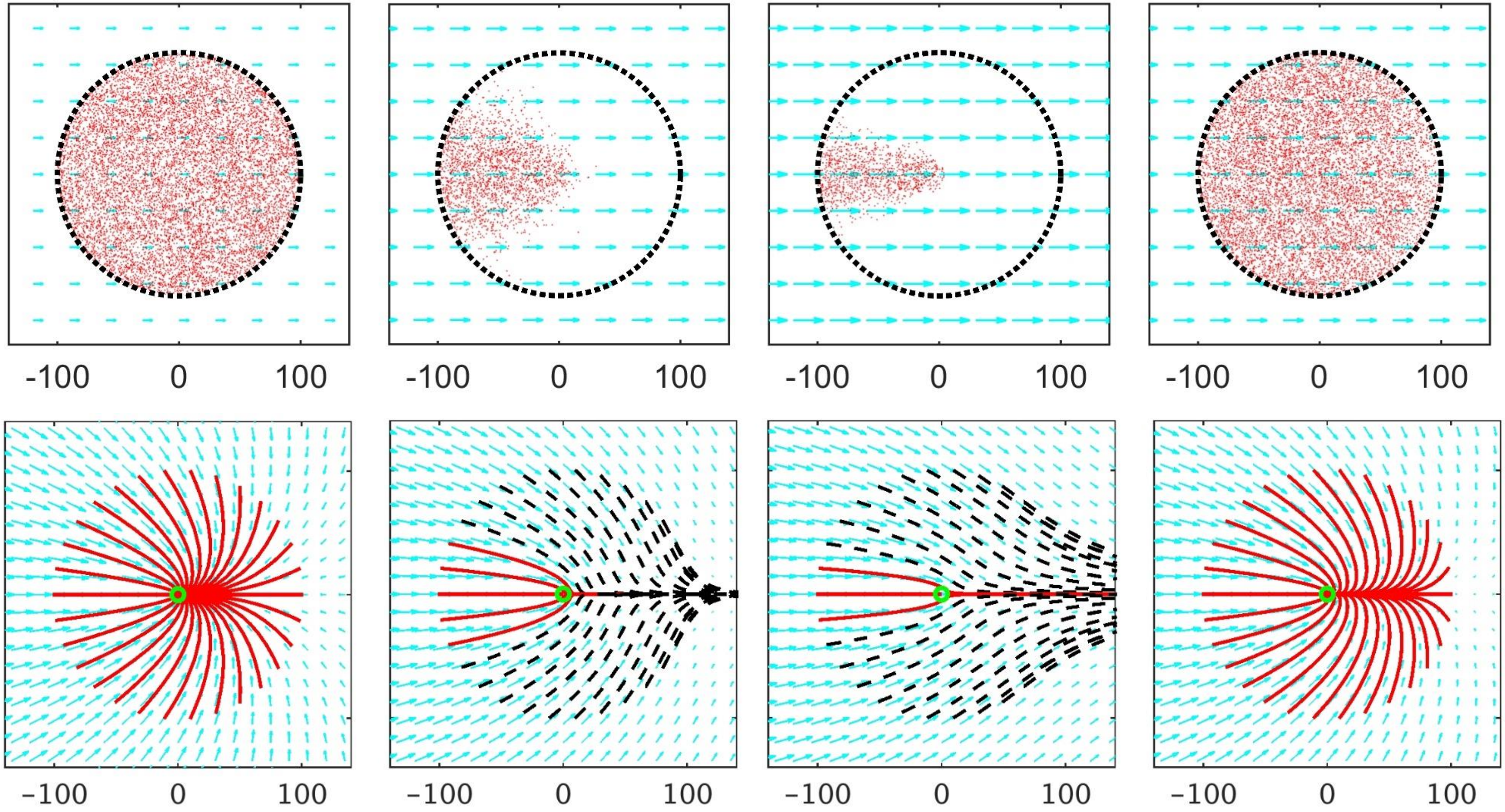
Flow is constant, $\mathbf{u} = (u_x, 0)$, and navigation is determined from the VM distributional choice. Gives…

$$\begin{aligned} \dot{x} &= u_x - c_1 \frac{x}{\sqrt{x^2+y^2}} - c_3 \frac{x}{x^2+y^2}\,; \\ \dot{y} &= -c_1 \frac{y}{\sqrt{x^2+y^2}} - c_3 \frac{y}{x^2+y^2}\,. \end{aligned}$$

$$c_1(k) := s\frac{I_1(k)}{I_0(k)}, \qquad c_2(k) := \frac{s^2}{2\lambda}\left(1 - \frac{I_2(k)}{I_0(k)}\right), \qquad c_3(k) := \frac{s^2}{\lambda}\left(\frac{I_2(k)}{I_0(k)} - \frac{I_1(k)^2}{I_0(k)^2}\right).$$

Goal Navigation: Idealised

Compare MC trajectories with earlier IBM simulations:



Goal Navigation: Idealised

Phase plane analysis reveals 2 or 3 steady states:

1. The target (i.e. the centre of the goal) - unstable node;
2. Upstream – saddle;
3. Downstream – stable node when it exists.

Goal Navigation: Idealised

Phase plane analysis reveals 2 or 3 steady states:

1. The target (i.e. the centre of the goal) - unstable node;
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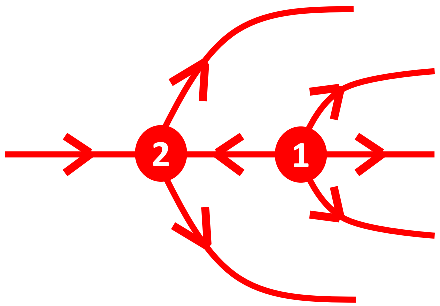
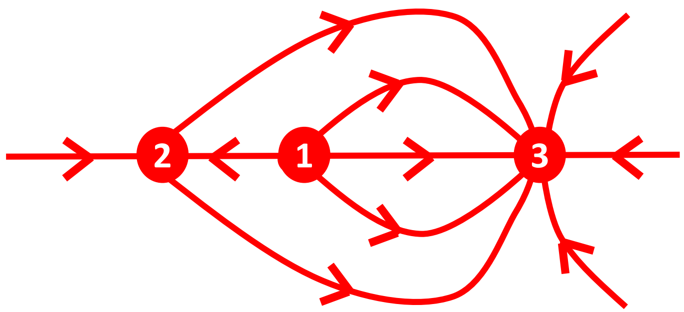
3. Downstream – stable node when it exists.

$$u_x > s\frac{I_1(k)}{I_0(k)}$$



Trajectories swept out.

$$u_x < s\frac{I_1(k)}{I_0(k)}$$



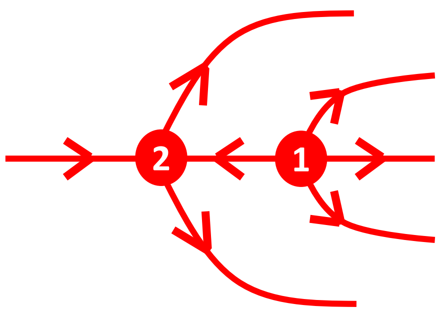
Trajectories attracted to 3

Goal Navigation: Idealised

Phase plane analysis reveals 2 or 3 steady states:

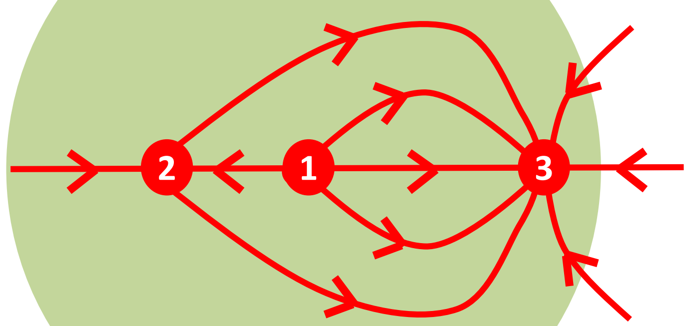
1. The target (i.e. the centre of the goal) - unstable node;
2. Upstream – saddle;
3. Downstream – stable node when it exists.

$$u_x > s\frac{I_1(k)}{I_0(k)}$$



Trajectories swept out.

$$u_x < s\frac{I_1(k)}{I_0(k)}$$



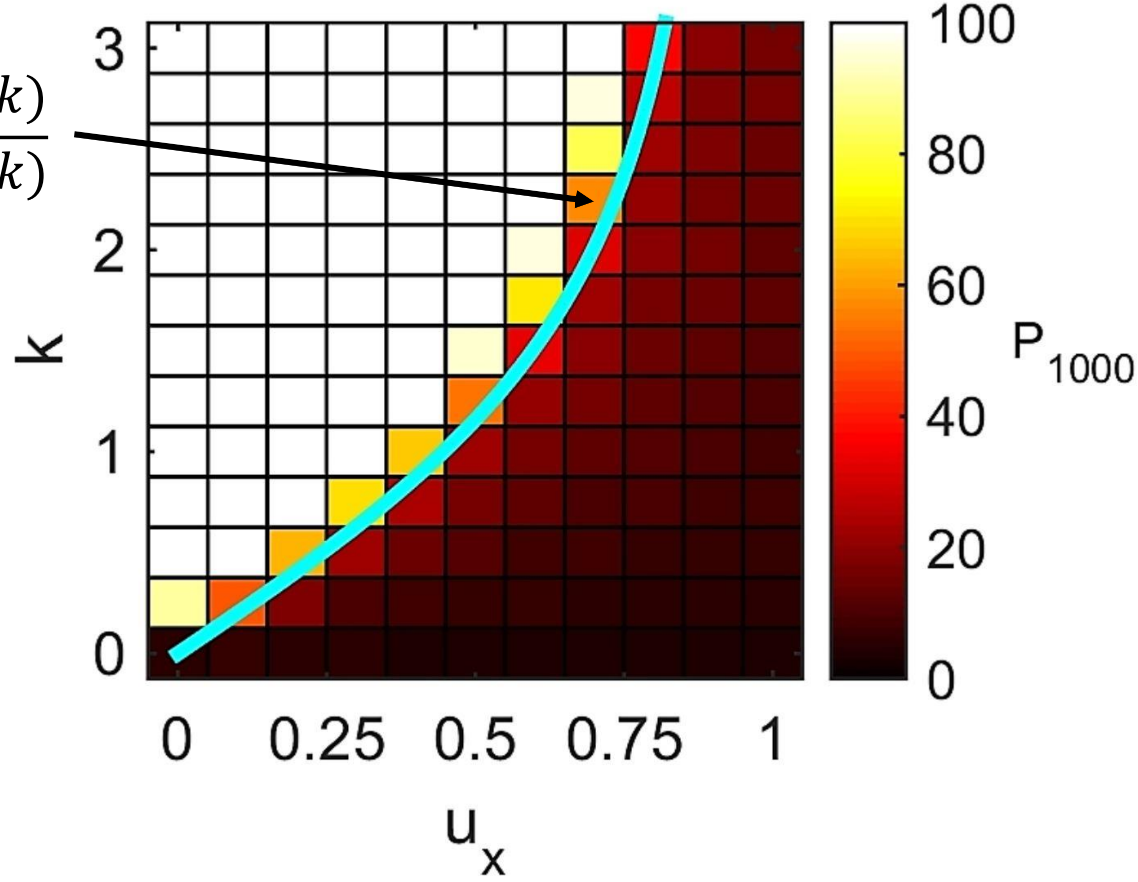
Trajectories attracted to 3

If (SS3) lies inside goal then we have eventual homing success

Goal Navigation: Idealised

Overlay on earlier parameter sweep suggests that this gives a good proxy for population success:

$$u_x = s\frac{I_1(k)}{I_0(k)}$$

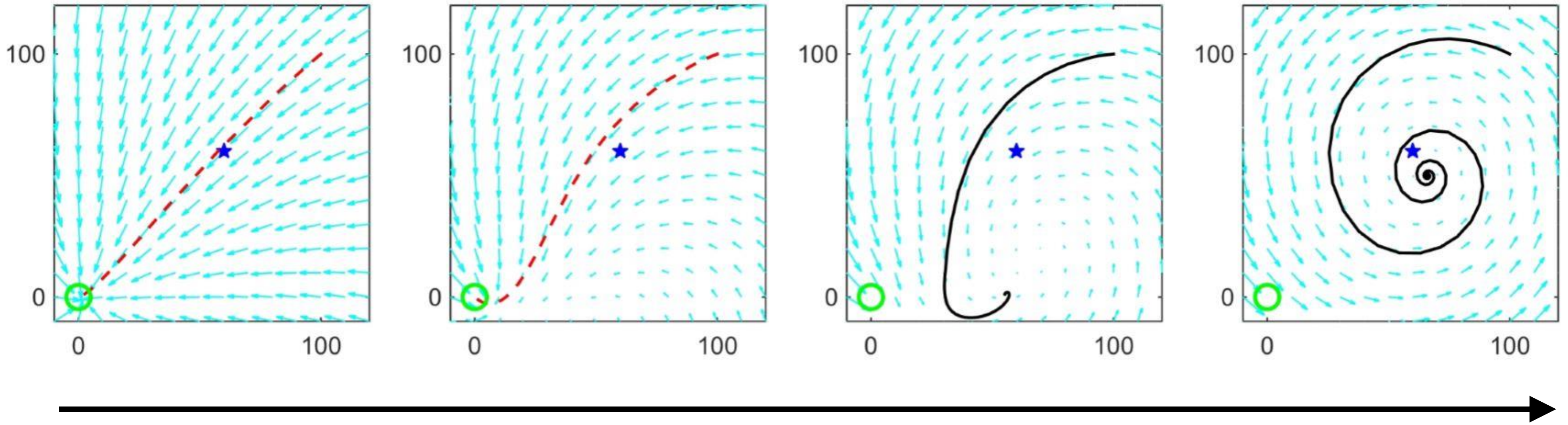


Goal Navigation: Idealised

Can also investigate more complicated flows: e.g. adding a "whirlpool" between the individual's starting location and the goal.

$$\dot{x} = \gamma(66 - 0.1x - y) - c_1 \frac{x}{\sqrt{x^2+y^2}} - c_3 \frac{x}{x^2+y^2}$$

$$\dot{y} = \gamma(x - 0.1y - 54) - c_1 \frac{y}{\sqrt{x^2+y^2}} - c_3 \frac{y}{x^2+y^2}$$



Increasing gamma

Goal Navigation: Ascension Island

- Turtles swim from South America coastal waters to Ascension Island to breed/lay eggs once every few years.
- Breeding season is from late December/January to June/July.

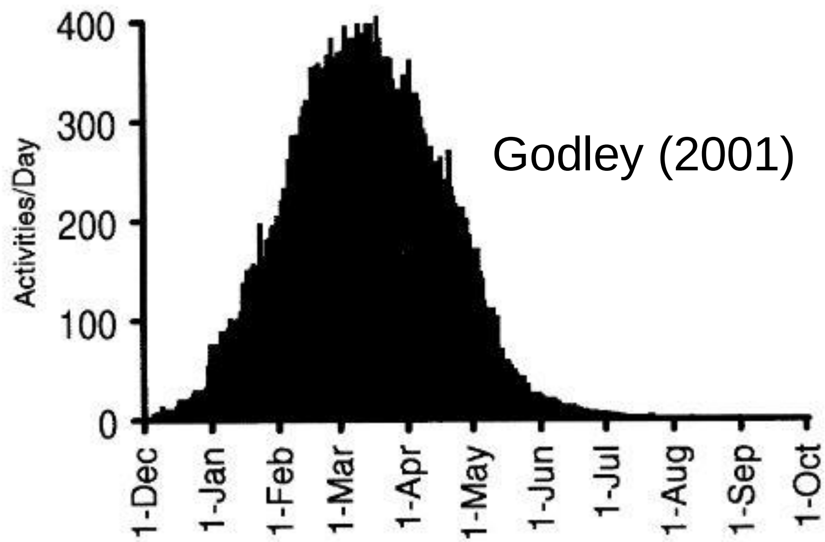


Fig. 2. Temporal distribution of nesting activities of green turtles on Ascension Island during the 1998/1999 season. For each beach, the

- Most females lay multiple clutches of eggs during the season.

Goal Navigation: Ascension Island

- Highly difficult to track full navigation pathway.
- Instead, females at AI are displaced and tracked.

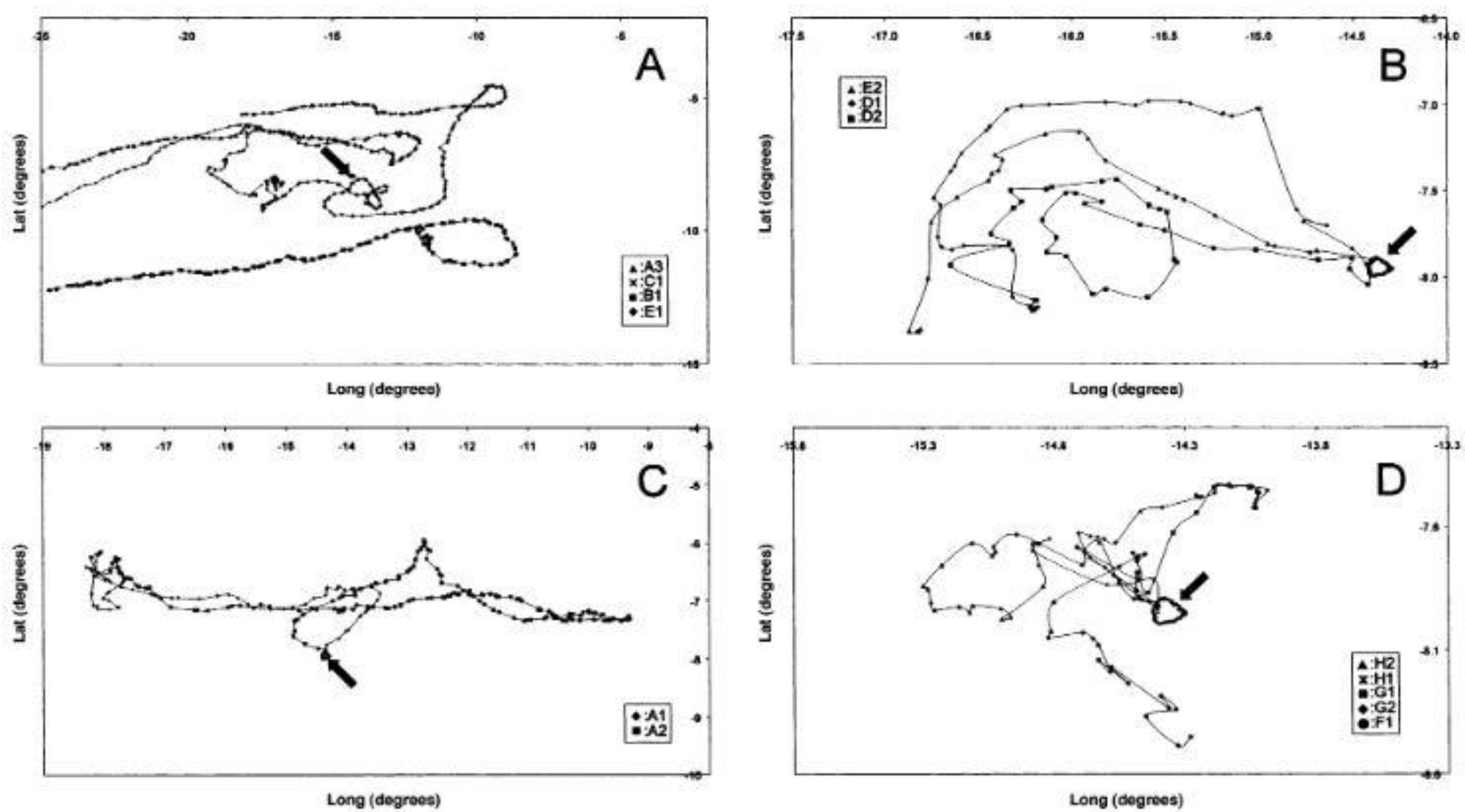
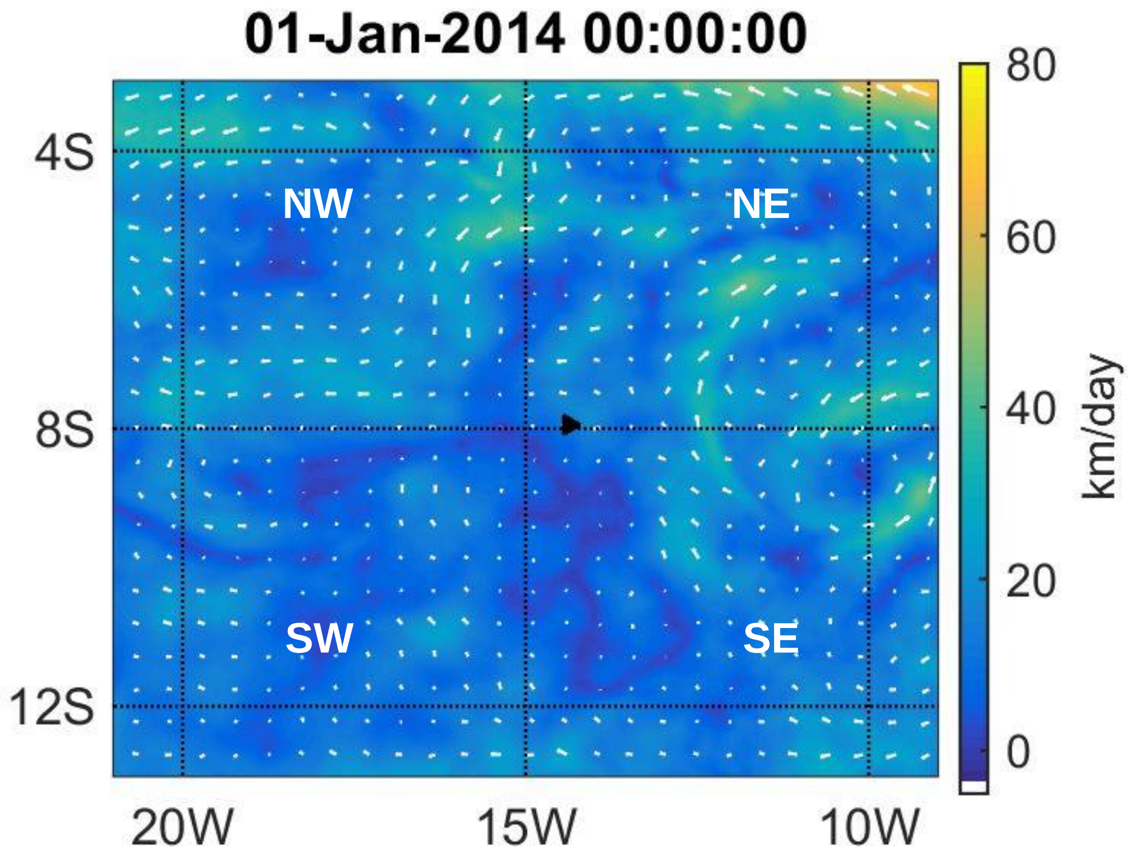


Fig. 2. Satellite tracks of adult female green turtles displaced by ship and released at sea at eight different sites around Ascension Island. (A) Female green turtles classified as searching for Ascension Island, but were not able to locate it, and (B–D) turtles

Goal Navigation: Ascension Island

- Consider the 2D ocean surface centred about Ascension Island (turtles spend the majority of the time at/near the surface).
- Simulate displacement experiments:

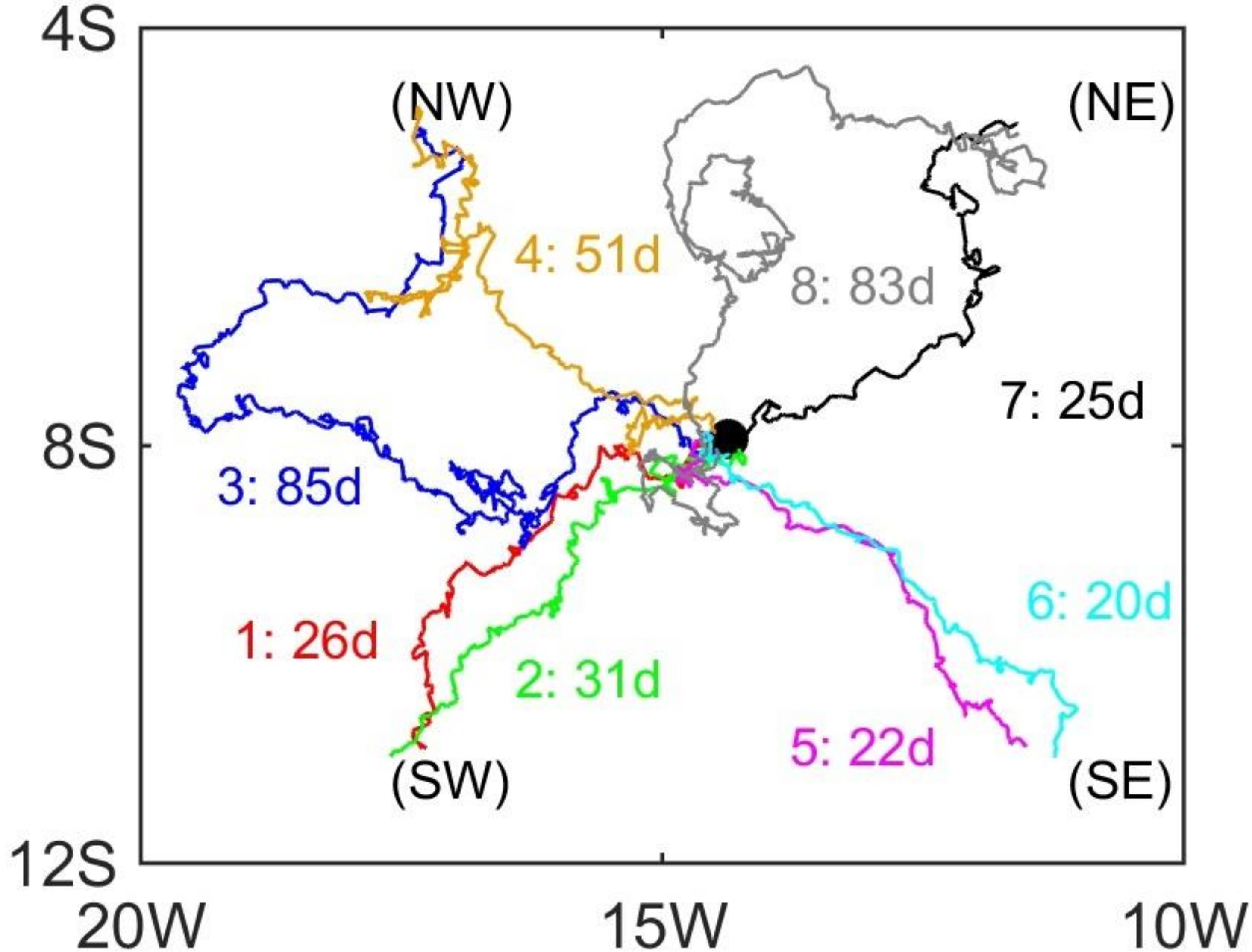


Goal Navigation: Ascension Island

- Consider the 2D ocean surface centred about Ascension Island (turtles spend the majority of the time at/near the surface).
- Simulate displacement experiments.
- Parameter set:
  - Active swimming speed *s = 0 - 80 km/day (i.e. ranges from a passive drifter to upper ends of energetically feasible limits)*
  - Turning rate 12 per day (roughly, once every couple of hours).
  - Navigational strength *k = 0 – 3 (i.e. ranges from a random mover to a "strong" navigator).*
- "Success" measures:
  - P_100: Population percentage that return within 100 days.
  - T_{1/2}: Time by which half the population has successfully returned.

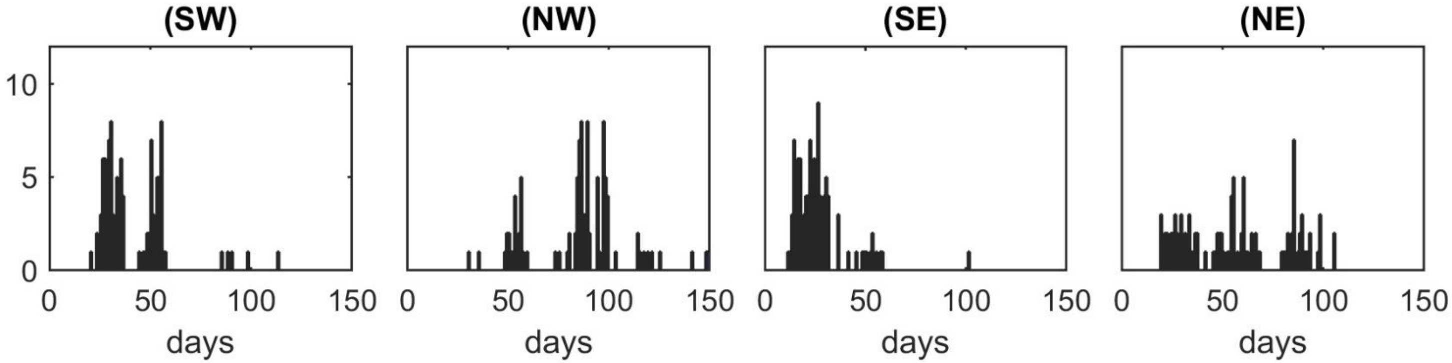
Goal Navigation: Ascension Island

s = 50 km/day, k = 1, Release date: 01/01/14



Goal Navigation: Ascension Island

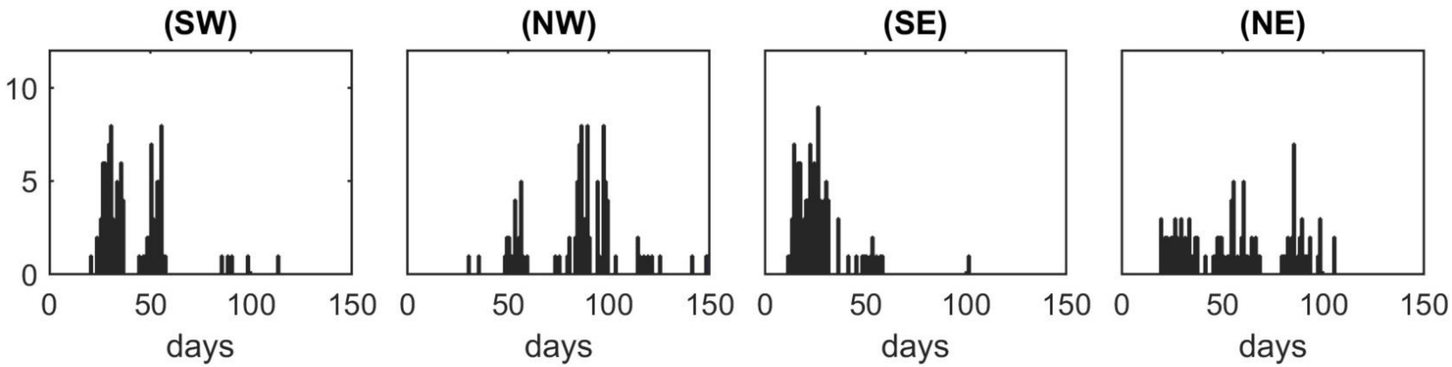
s = 50 km/day, k = 1, Release date: 01/01/14



(SW)	(NW)	(SE)	(NE)
$T_{1/2}$ = 35.5 days	$T_{1/2}$ = 86.9 days	$T_{1/2}$ = 23.1 days	$T_{1/2}$ = 55.6 days
P_{100} = 99 %	P_{100} = 89 %	P_{100} = 99 %	P_{100} = 98 %

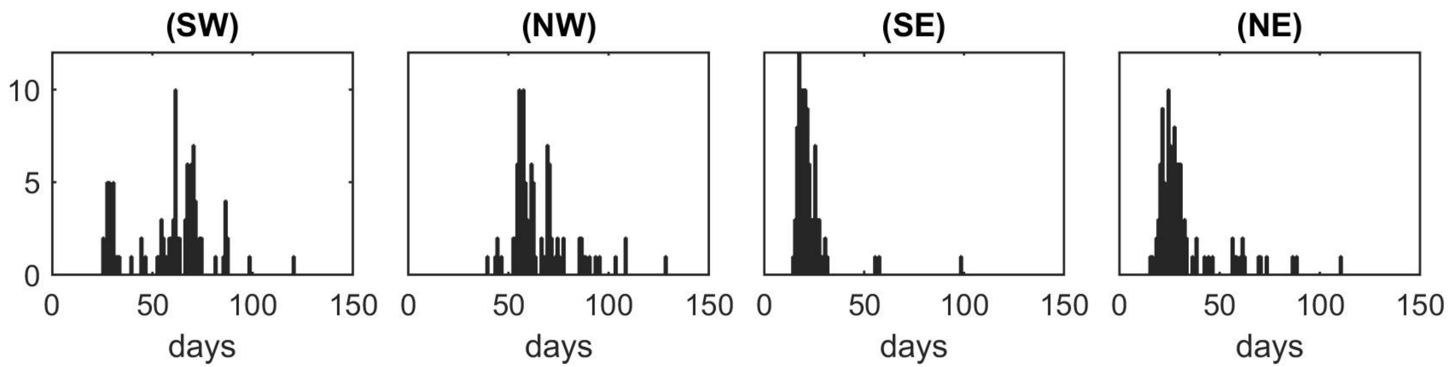
Goal Navigation: Ascension Island

s = 50 km/day, k = 1, Release date: 01/01/14



(SW)	(NW)	(SE)	(NE)
$T_{1/2}$ = 35.5 days	$T_{1/2}$ = 86.9 days	$T_{1/2}$ = 23.1 days	$T_{1/2}$ = 55.6 days
P_{100} = 99 %	P_{100} = 89 %	P_{100} = 99 %	P_{100} = 98 %

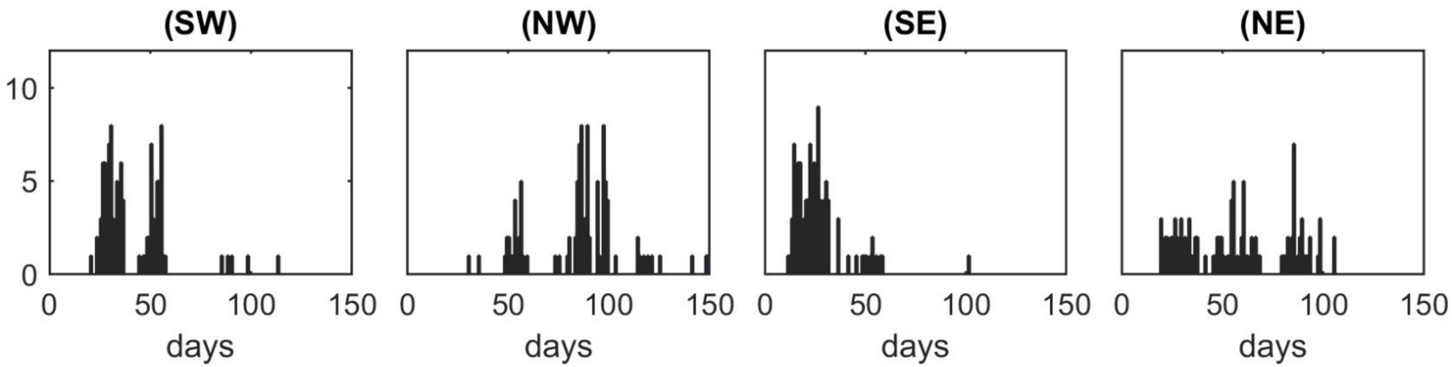
s = 50 km/day, k = 1, Release date: 29/01/14



(SW)	(NW)	(SE)	(NE)
$T_{1/2}$ = 61.6 days	$T_{1/2}$ = 60 days	$T_{1/2}$ = 20.2 days	$T_{1/2}$ = 27.2 days
P_{100} = 99 %	P_{100} = 96 %	P_{100} = 100 %	P_{100} = 99 %

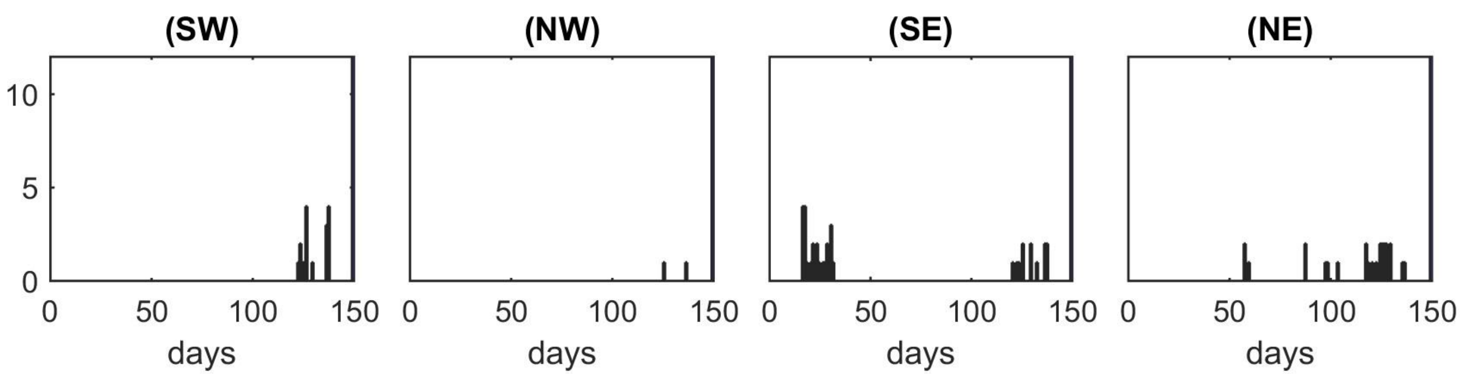
Goal Navigation: Ascension Island

s = 50 km/day, k = 1, Release date: 01/01/14



$T_{1/2}$ = 35.5 days	$T_{1/2}$ = 86.9 days	$T_{1/2}$ = 23.1 days	$T_{1/2}$ = 55.6 days
P_{100} = 99 %	P_{100} = 89 %	P_{100} = 99 %	P_{100} = 98 %

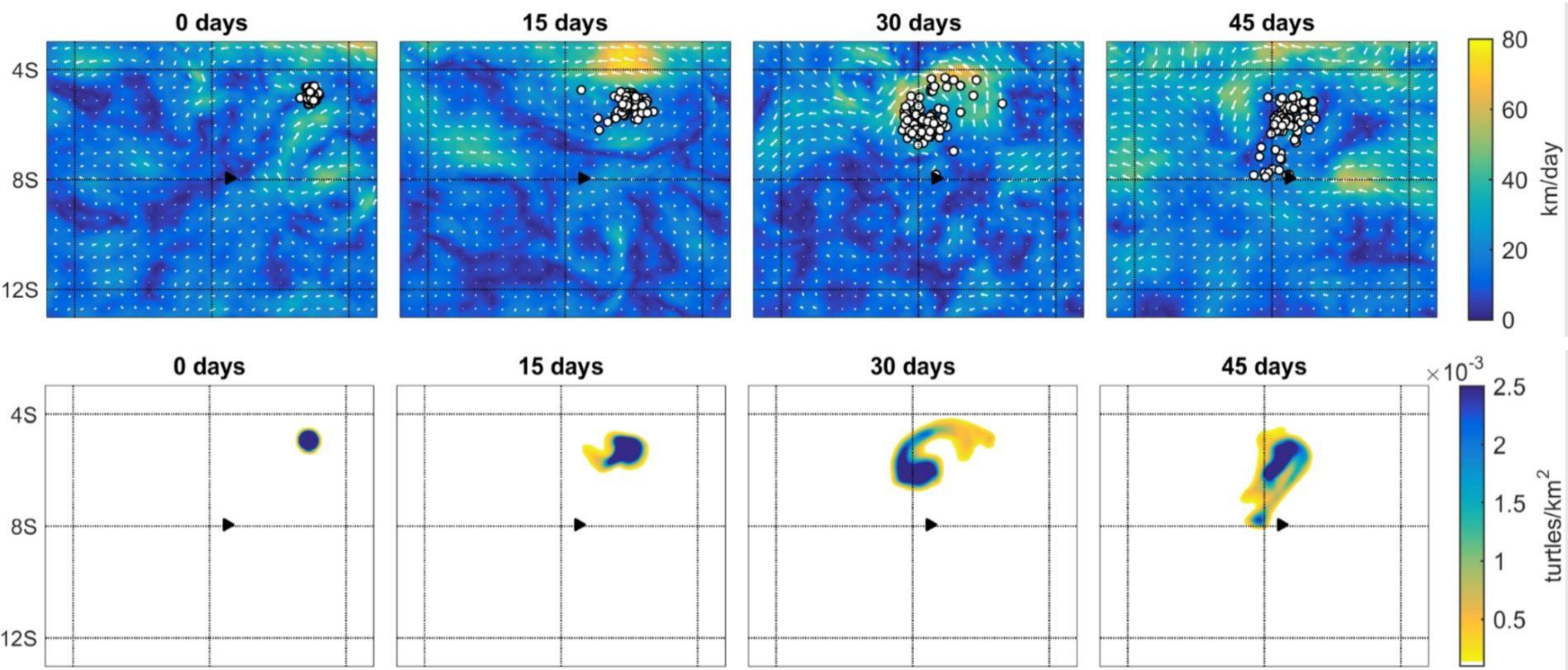
s = 30 km/day, k = 1, Release date: 01/01/14



$T_{1/2}$ > 150 days	$T_{1/2}$ > 150 days	$T_{1/2}$ > 150 days	$T_{1/2}$ > 150 days
P_{100} = 0 %	P_{100} = 0 %	P_{100} = 24 %	P_{100} = 7 %

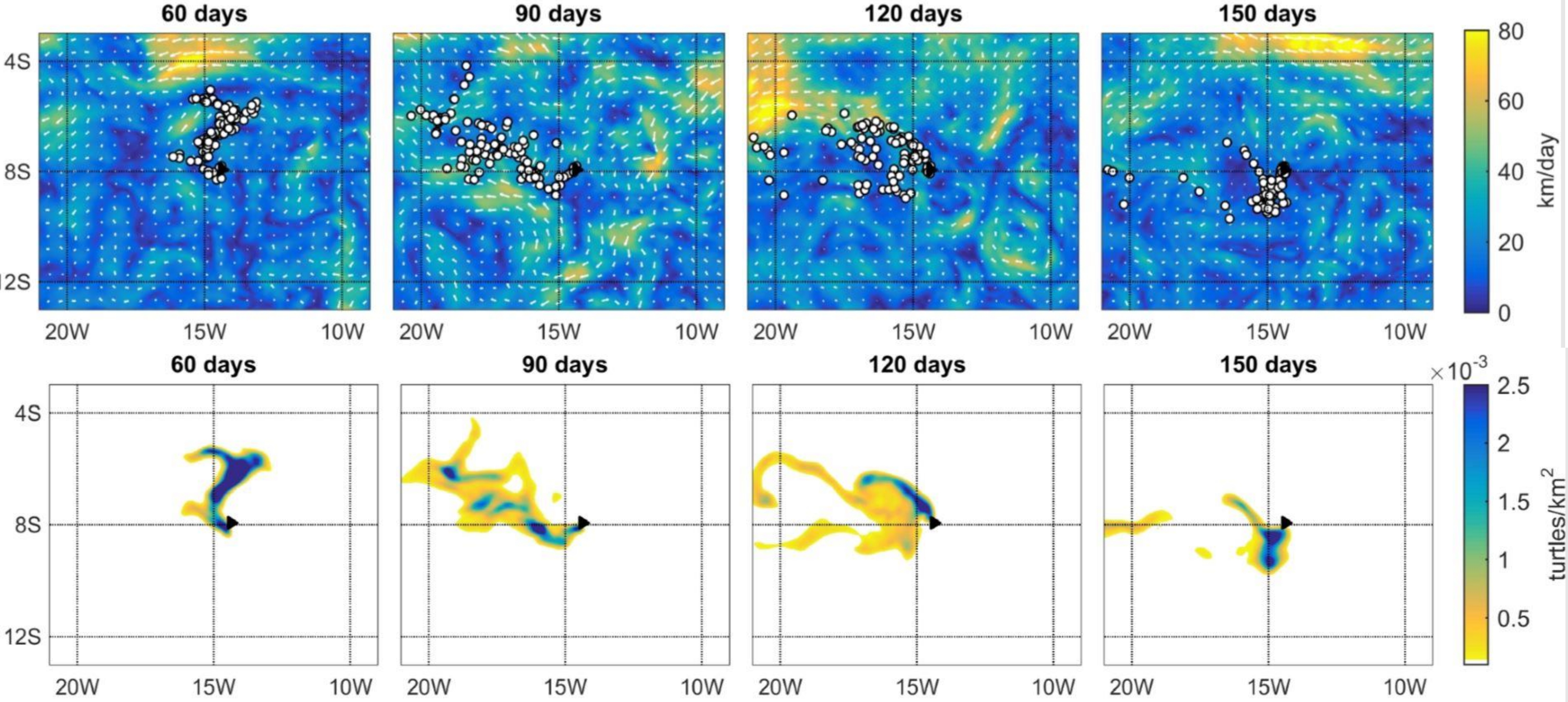
Goal Navigation: Ascension Island

Comparison between IBM and macroscopic model:



Goal Navigation: Ascension Island

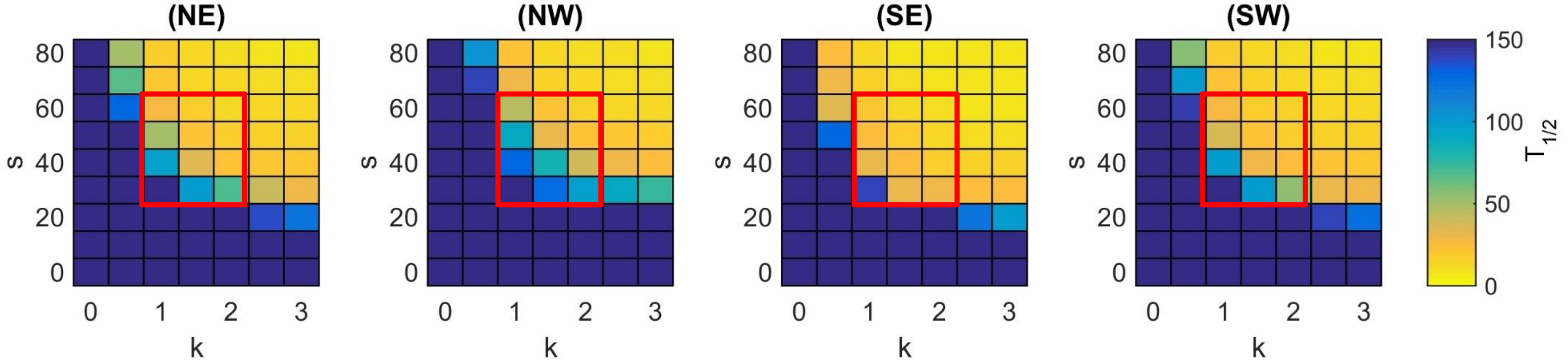
Comparison between IBM and macroscopic model:



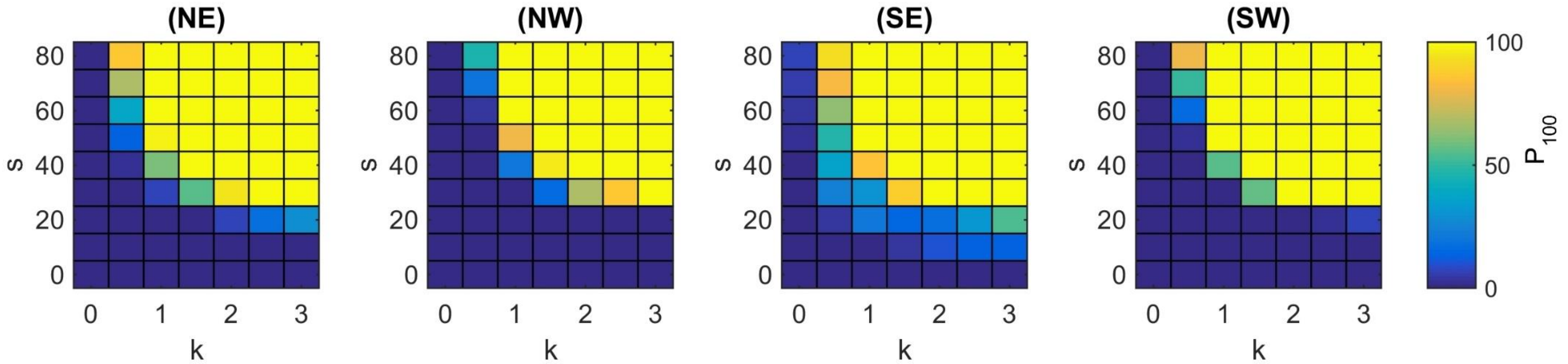
Simulations: IBM takes order of an hour while the macroscopic model takes order of minutes….

Goal Navigation: Ascension Island

Sweep across mean speed/navigating strength parameter space:



(a)



(b)

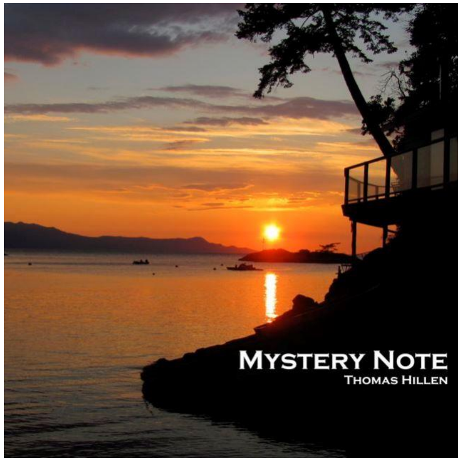
Summary

- Multiscale framework for movements in flowing environments. Beyond navigation, potential applications in:
  - Modelling fish (egg, larval and adult stages) distributions
  - Insect infestations etc.
- Current work: more detailed modelling of different hypotheses for turtle navigation.
- Challenges: efficient and stable solvers for anisotropic diffusion equations (also has important consequences in image processing).
- Challenges: “Mean free passage time - what is the expected time for individuals to reach a target?” Solved for isotropic diffusion in the absence of flow, but not for the more complicated model here.

References/Thanks

Thanks to my collaborator: mathematician and jazz pianist:

Thomas Hillen (University of Alberta)



- K. J. Painter, T. Hillen (2015). Navigating the flow: modelling animal navigation in flowing environments. Submitted to J. Roy. Soc. Interface.
- T. Hillen, K. J. Painter (2013). Transport and Anisotropic Diffusion Models for Movement in Oriented Habitats. In “Dispersal, Individual Movement and Spatial Ecology”. Springer. Book Chapter.