Abstracts

Short courses

Erwan Faou
Introduction to weak error analysis for stochastic differential equations

The goal of this course is to present some results concerning the numerical approximations of stochastic differential equations. We will discuss the link between SDEs and partial differential equations, and discuss convergence results. Several examples will be given, in particular in molecular dynamics where dynamics are constructed to satisfy some ergodic properties that can be reproduced numerically.

Christian Lubich
Modulated Fourier expansions for continuous and discrete oscillatory systems

This course reviews some of the phenomena and theoretical results on the long-time energy behaviour of continuous and discretized oscillatory systems that can be explained by modulated Fourier expansions: long- time preservation of total and oscillatory energies in oscillatory Hamil- tonian systems and their numerical discretisations, near-conservation of energy and angular momentum of symmetric multistep methods for ce- lestial mechanics, metastable energy strata in nonlinear wave equations, and long-time stability of plane wave solutions of nonlinear Schroedinger equations. We discuss what modulated Fourier expansions are and what they are good for.

Much of the presented work was done in collaboration with Ernst Hairer. Some of the results on modulated Fourier expansions were obtained jointly with David Cohen and Ludwig Gauckler.

Senior participants

Jonathan Sherratt
Spatiotemporal patterns behind invasions in reaction-diffusion equations

Oscillatory reaction-diffusion equations arise as models of a variety of physical and biological phenomena, including chemical reactions and predator-prey interactions. Invasions in such systems, for example predators invading prey, typically generate periodic spatiotemporal patterns. Often these occur only as a transient, although one that is a long term feature of the solution since it moves with the invasion. In this lecture I will explain this seemingly complex behaviour via the theory of absolute and convective instability, showing how to calculate the width of the band of periodic patterns. I will end by discussing the implications of my results for predator-prey systems.

David Siska
Convergence of tamed Euler schemes for a class of stochastic evolution equations

We prove stability and convergence of a full discretization for a class of stochastic evolution equations with super-linearly growing operators appearing in the drift term. This is done by using the recently developed tamed Euler method, which employs a fully explicit time stepping, coupled with a Galerkin scheme for the spatial discretization. (joint work with I. Gyongy and S. Sabanis)

Claudia Wulff
A-stable Runge-Kutta time-semidiscretizations of semilinear Hamiltonian PDEs

We consider semilinear Hamiltonian evolution equations, for example the semilinear wave equation with periodic or Dirichlet boundary conditions. When the linear part of the evolution equation is skew symmetric and the nonlinear part is smooth on a suitable scale of Hilbert spaces we show that its flow and a A stable Runge Kutta time discretization of it are smooth as maps from an open subset of the highest rung of the scale into the lowest rung, and obtain full order convergence results for the time-discretization. Then we prove fractional order convergence results for the trajectory error of the time-semidiscretization in the case of non-smooth initial data, and show that the energy error, for nonsmooth initial data, has much higher order than the trajectory error. We also briefly discuss fractional order convergence results for exponential Runge-Kutta methods. Finally, if time allows, we show that for analytic data symplectic A-stable RK one-step methods conserve a modified energy up to an exponentially small error. In contrast to ODEs, this does not imply long time approximate energy conservation of numerical trajectories. The situation is analogous to corresponding results on approximate energy conservation of rapidly forced PDEs (see Matthies/Scheel 2003).

This is joint work with Marcel Oliver and Chris Evans.

Student talks

Markus Ableidinger
Splitting integrators for the Landau-Lifshitz-Gilbert equation

The stochastic Landau-Lifshitz-Gilbert (SLLG) equation describes the magnetisation vector field of a nano- or microscale ferromagnetic material occupying a bounded domain $D \subset \mathbb{R}^n$, $n = 2, 3$, where the so-called effective field is subject to thermal fluctuations. Given a complete probability space $(\Omega, \mathcal{F}, P)$ with filtration $\mathcal{F}_t$ and a Hilbert space $\mathcal{K}$, the SLLG takes the form

$$dm(t,x) = m(t,x) \times [H_{\text{eff}}(m) - \alpha m(t,x) \times H_{\text{eff}}(m)]\, dt + \nu m(t,x) \times \circ dW(t,x)$$

with

$$\begin{aligned} \partial_n m(t,x) &= 0 \qquad \forall (t,x) \in \mathbb{R}^+ \times \partial D \\ m(0,x) &= m_0(x) \qquad \forall x \in D. \end{aligned}$$

W is a $\mathcal{K}$–valued Wiener process, $\alpha, \nu \in \mathbb{R}$ and $\circ$ denotes a Stratonovich-type integral. An important requirement for a successful numerical treatment is that the numerical methods respect the qualitative behaviour of the SLLG, for example the conservation of the modulus

$$|m(t,x)| = 1 \qquad\qquad \forall (t,x).$$

In this talk we will discuss numerical methods for solving the corresponding system of SODEs after spatial discretisation. Our main focus will be on splitting integrators which allow a good approximation of the deterministic geometric properties of the

SLLG under reasonable computational effort.

Arianna Bianchi
A mathematical model for lymphangiogenesis in wound healing

Several studies suggest that one possible cause of impaired wound healing is the failed or insufficient lymphangiogenesis, that is the formation of new lymphatic capillaries.
Although many mathematical models have been done to describe the formation of blood capillaries (angiogenesis), very few have been proposed for this phenomenon involving the regeneration of the lymphatic vessels' network. Moreover, lymphangiogenesis is a quite different process from angiogenesis, occurring at different times and in a different manner.
Here a model of five ordinary differential equations is presented to describe the formation of lymphatic capillaries after a skin wound. The variables represent different cell densities and growth factor concentrations, and when possible the parameters are estimated from real biological data. A simulation of the model is run to compare the predicted results with the ones expected in a real-life situation.

Antoine Choffrut
Rayleigh-Benard convection: physically relevant a priori estimates

Raffaele d'Ambrosio
Structure preserving numerical methods for differential equations

It is the purpose of this talk to analyze the nearly conservative behaviour of multi-value methods for the numerical solution of ordinary di?fferential equations and, in particular, of Hamiltonian and oscillatory problems. We provide long-term error estimates for multi-value methods, in order to understand if such methods can be assumed as good candidates for an excellent long-term invariants preservation. In particular, a backward error analysis is presented, which permits to get sharp estimates for the parasitic solution components and for the error in the Hamiltonian. We also consider structure preservation properties in the numerical solution of oscillatory problems based on partial di?fferential equations, typically modelling oscillatory biological systems, whose solutions oscillate both in space and in time. Special purpose numerical methods able to accurately retain the oscillatory behaviour are presented.

James–Michael Leahy
On degenerate stochastic evolution equations driven by jump processes

The aim of this talk is to present an existence and uniqueness result for solutions of degenerate linear stochastic evolution equations driven by jump processes. This result generalizes a classical existence and uniqueness theorem for equations with continuous martingale noise that was established by N. V. Krylov and B. L. Rozovskii in 1980. We approach the equations from the variational framework and apply the method of vanishing viscosity in a Hilbert scale. As an application, we derive the existence and uniqueness of solutions of degenerate parabolic linear stochastic integro-differential equations in the Sobolev scale. Such equations play an important role in non-linear filtering of semimartingales. We shall also briefly report on an extension to L_p spaces. This work was done in collaboration with Remigijus Mikulevicius (University of Southern California).

Michael Tsardakas
Mathematics of crime

Using mathematical methods to understand and model crime is a relatively recent idea that has drawn considerable attention from researchers during the last five years. From the plethora of models that have been proposed perhaps the most successful one has been a diffusion-type differential-equations model that describes how the number of criminals evolves in a specific area. We propose a more detailed form of this model that allows for two distinct criminal types representing major and minor crime. Additionally, we examine a stochastic variant of the model that represents more realistically the "generation" of new criminals. Numerical solutions from both models are presented and compared with actual crime data for the Greater Manchester area. Agreement between simulations and actual data is satisfactory. A preliminary statistical analysis of the data also supports the models potential to describe crime. (joint work with Prof. Andrew Lacey)

Jin-Han Xie
A coupled model of the interactions between near-inertial waves and mesoscale mean flow in the ocean

We derive a new model of the interactions between near-inertial waves (NIWs) and balanced motion in the ocean using Generalised Lagrangian Mean theory and Whitham averaging. In its simplest form, the model couples the wellknown Young-Ben Jelloul model of NIWs with a quasi- geostrophic model of the balanced motion. The model is Hamiltonian and conserves both energy and wave action. This coupled model provide a novel mechanism of energy transfer from the mean flow to the NIW in the real ocean, and numerical simulations of two simplified 2D model shed light on physics of NIW-mean flow interactions. (with Jacques Vanneste)