Schedule

Thursday	10:00	*Coffee and registration*
	10:30	Wolfgang Wendland (Stuttgart)
	11:10	Chiara Guardasoni (Parma)
	11:50	Ceyhun Özdemir (Graz)
	12:30	*Lunch break*
	14:00	Alexey Chernov (Oldenburg)
	14:40	Thomas Hartmann (Ulm)
	15:20	*Coffee break*
	16:00	Dirk Praetorius (Wien)
	16:40	Gregor Gantner (Wien / Amsterdam)
Friday	9:00	Norbert Heuer (Santiago de Chile)
	9:40	Thomas Führer (Santiago de Chile)
	10:20	*Coffee break*
	11:00	Luca Desiderio (Parma)
	11:40	Giulia Di Credico (Innsbruck / Parma)
	12:20	*Lunch break*
	14:00	Joachim Gwinner (München)
	14:40	Lothar Banz (Salzburg)
	15:20	*Coffee break*
	16:00	Stefan Sauter (Zürich)
	16:40	Francesco Florian (Zürich)
	18:30	*Workshop dinner:* Weisses Rössl, Kiebachgasse 8
Saturday	9:00	Markus Melenk (Wien)
	9:40	David Wörgötter (Wien)
	10:20	*Coffee break*
	10:50	Daniel Seibel (Saarbrücken)
	11:30	Carsten Carstensen (Berlin)

The workshop takes place at the *Campus Technik* of the University of Innsbruck: Lecture Hall HSB1, Technikerstrasse 13b, 6020 Innsbruck.

Abstracts: Thursday

Wolfgang Wendland (Stuttgart)
Propagation of acoustic waves in a thermo-electro-magneto-elastic solid

(with G.C. Hsiao, Univ. of Delaware, USA) We are concerned with a time–dependent transmission problem for a thermo–electric–magneto–elastic solid immersed in an inviscid and compressible fluid. This problem can be treated by a boundary–field equation method, provided an appropriate scaling factor is employed. Based on estimates of variational solutions in the Laplace–transformed domain we obtain with explicit inversions of the variational solution a convolutional form in the Laplace–transformed domain and also the time–dependent representation.

Chiara Guardasoni (Parma)
Recent advances in 2D elastodynamics by time-domain energetic boundary element methods

(with A. Aimi, G. Di Credico, H. Gimperlein, G. Speroni) The contribution aims at illustrating some recent advances in the application of the Energetic Boundary Element Method (EBEM) to the numerical resolution of 2D elastodynamic problems. From the initial works it has been clear that the energetic weak approach to boundary integral formulations gives a good theoretical setting for the investigation of elastodynamic wave propagation. The method is based on a boundary integral representation formula for the differential problem solution, then a weak formulation linked to the energy of the system is applied in order to achieve, in the approximation phase, accurate and stable numerical results. However, the extension of the EBEM implementation is not straightforward since the computation of linear system entries requires sophisticated numerical strategies depending on the problem at hand (bounded or an unbounded domain equipped with Dirichlet and/or Neumann conditions) and on the integral formulation used to represent the solution. Numerical results coming from weak coupling of boundary conditions to the full integral representation formula will be presented and discussed.

Ceyhun Özdemir (Graz)
3D Time-domain boundary element method for the wave equation with stochastic boundary conditions

In this talk, we introduce the homogeneous wave equation with Dirichlet and acoustic boundary conditions depending on a probablity. At first, we have a look on the deterministic Dirichlet problem and derive a time-domain boundary element method (TDBEM). With a specific choice of ansatz and test functions, we apply the marching-on-in time (MOT) method. We extend this approach to the probabilistic Dirichlet problem via a polynomial chaos discretization and present numerical experiments. Further we consider a probabilistic acoustic problem and derive a stochastic TDBEM system. We apply an MOT method again and finish the talk with numerical examples.

Alexey Chernov (Oldenburg)
Bayesian parameter identification of impedance boundary condition for Helmholtz problems

(with Nick Wulbusch, Reinhild Roden and Matthias Blau) We address the problem of identifying the acoustic impedance of a wall surface from noisy pressure measurements in a closed room with a Bayesian approach. The room acoustics is modelled by the interior Helmholtz equation with impedance boundary conditions. The aim is to compute moments of the acoustic impedance to estimate a suitable density function of the impedance coefficient. For the computation of moments we use ratio estimators and Monte-Carlo sampling. We consider two different experimental scenarios. In the first scenario the noisy measurements correspond to a wall modelled by impedance boundary conditions. In this case the Bayesian algorithm uses a model that is (up to the noise) consistent with the measurements and therefore reaches very good accuracy observed in our numerical experiments. In the second scenario the noisy measurements come from a coupled acoustic-structural problem corresponding to the case of a wall made of glass, whereas the Bayesian algorithm still uses a model with impedance boundary condition. In this case the parameter identification model is inconsistent with the measurements and therefore is not capable to represent them well. Nonetheless, for particular frequency bands the Bayesian algorithm identifies estimates with relatively high likelihood. Outside these frequency bands the algorithm fails. We discuss the results of both examples and possible reasons for the failure of the latter case for particular frequency values.

Thomas Hartmann (Ulm)
A method for determining the sound parameters from a solution of the Helmholtz equation in the time domain

For the efficient development of machine and vehicle components, such as e.g. gearboxes, the calculation of the expected noise is essential. Vibrations of non-linear components or transient operating conditions are mainly simulated with multi-body simulation programs in the time domain, while the sound is calculated with separate software in the frequency domain. In order to avoid transformation errors and to save resources, the sound calculation is carried out directly in the time domain instead. By applying the boundary element method, the number of degrees of freedom to be considered can be extremely reduced, the airborne noise can be calculated parallel to the structure simulation and analyzed directly after each simulation time step.

Dirk Praetorius (Wien)
Adaptive FEM with quasi-optimal cost for nonlinear PDEs

We consider nonlinear elliptic PDEs with strongly montone nonlinearity. We apply an adaptive finite element method, which steers the linearization as well as the iterative solution of the arising linear finite element systems. We prove that the proposed algorithm guarantees full linear convergence (i.e., linear convergence in each step, independently of the algorithmic decision for mesh-refinement, linearization, or algebraic solver step). For sufficiently small adaptivity parameters, this allows to guarantee optimal convergence with respect to the overall computational work (i.e., the computational time).
The talk is based on joint work [1, 2, 3].

[1] G. Gantner, A. Haberl, D. Praetorius, S. Schimanko: Rate optimality of adaptive finite element methods with respect to the overall computational costs, Mathematics of Computation, 90 (2021), 2011-2040.

[2] A. Haberl, D. Praetorius, S. Schimanko, M. Vohralik: Convergence and quasi-optimal cost of adaptive algorithms for nonlinear operators including iterative linearization and algebraic solver, Numerische Mathematik, 147 (2021), 679-725.

[3] P. Heid, D. Praetorius, T. Wihler: Energy contraction and optimal convergence of adaptive iterative linearized finite element methods, Computational Methods in Applied Mathematics, 21 (2021), 407-422.

Gregor Gantner (Wien / Amsterdam)
Adaptive space-time BEM for the heat equation

In this talk, we consider the space-time boundary element method for the heat equation with prescribed initial and Dirichlet data. One often mentioned advantage of simultaneous space-time methods is their potential for fully adaptive refinement to resolve singularities local in both space and time. To this end, we propose a residual a posteriori error estimator similar to the Faermann estimator of the stationary case that is a lower bound and, up to weighted L_2-norms of the residual, also an upper bound for the unknown BEM error. The possibly locally refined meshes are assumed to be prismatic, i.e., their elements are tensor-products $J \times K$ of elements in time J and space K. While the results do not depend on the local aspect ratio between time and space, assuming the scaling $|J| \simeq \mathrm{diam}(K)^2$ for all elements and using Galerkin BEM, the estimator is shown to be efficient and reliable without the additional L_2-terms. In the considered numerical experiments on two-dimensional domains in space, the estimator seems to be equivalent to the error, independently of these assumptions. In particular for adaptive anisotropic refinement, both converge with the best possible convergence rate.

Abstracts: Friday

Norbert Heuer (Santiago de Chile)
Discretization of traces of some fourth-order problems

In recent years we have studied several mechanical models of thin structure deformation, and their discretization by discontinuous Petrov-Galerkin methods. In this context, ultra-weak variational formulations are attractive. Such formulations consider field variables to be no more than L2-regular so that test functions carry all the derivatives. The underlying integrations by part generate trace operators and it turns out that they reflect all the regularity properties of the underlying model problem. Not surprisingly, L2-field variables are easy to discretize but trace variables are much harder to come by, specifically in the case of thin structure models, or higher order PDEs in general.
In this talk we discuss some of the details of such trace operators and their discretization, and illustrate the application of our results to boundary element approximations of the biharmonic problem.
This talk is based on joint research with Thomas Führer (UC, Chile), Antti Niemi (U Oulu, Finland), and Alexander Haberl (TU Vienna, Austria).
The support of ANID through Fondecyt project 1190009 is gratefully acknowledged.

Thomas Führer (Santiago de Chile)
Multilevel decompositions and norms in negative order Sobolev spaces

We discuss multilevel norms that allow the efficient evaluation of negative order Sobolev norms for (discontinuous) polynomial functions. The results involve quasi-interpolation operators which can also be used to define local projection operators. Our findings allow to show stability of multilevel subspace decomposition of piecewise constants which directly lead to optimal multilevel diagonal scaling preconditioners. Our proposed decomposition does not depend on the number of levels and all our results are valid on uniform as well as adaptively generated meshes in arbitrary space dimensions. Applications include the preconditioning of boundary element methods.

Luca Desiderio (Parma)
Energetic boundary element method for 3D wavefield modelling

(with A. Aimi, S. Dallospedale, H. Gimperlein, C. Guardasoni) We consider elastodynamic (vector) wave equation, defined in unbounded domains external to 3D bounded ones, endowed with null initial conditions and with a Dirichlet condition on the boundary. For its numerical solution, we reformulate the original differential problem in terms of a space-time Boundary Integral Equation (BIE) and then we employ a weak formulation linked to the energy of the system in order to achieve, in the approximation phase by means of a Galerkin-type Boundary Element Method (BEM), accurate and stable numerical results. This approach, called Energetic BEM (EBEM), leads to a linear system whose matrix has Toeplitz lower-triangular block structure, that allows the acceleration of the solution phase. As a direct consequence of the flexibility of the EBEM, a large body of literature has risen after the pioneering paper [1], to witness its capabilities to simulate 3D acoustic [2] and 2D elastodynamic [3] wave propagation in semi-infinite or infinite media. However, the extension of the EBEM to 3D elastic problems is not straightforward, since the energetic full space-time discretization requires double integration both in space and in time. Since a key ingredient for the success of the EBEM is the efficient and accurate evaluation of all the involved integrals, the selected formulation could be quite challenging in large scale applications. Nevertheless, if standard (constant) time and shape functions are employed, the double integration in time can be performed analytically and one is left with the task of evaluating double space integrals, whose integration domains are generally delimited by the wavefronts of the primary and the secondary waves. In order to exactly detect this latter, and consequently to preserve the stability properties of the EBEM, we choose boundary meshes made by triangular elements with straight sides and we propose an ad-hoc numerical integration scheme, tailored for the correct domain of integration.

[1] A. Aimi, M. Diligenti, C. Guardasoni, I. Mazzieri, S. Panizzi, An energy approach to spacetime Galerkin BEM for wave propagation problems, Int. J. Numer. Meth. Engng., 80 (2009), pp. 1196–1240.

[2] A. Aimi, M. Diligenti, A. Frangi, C. Guardasoni, Neumann exterior wave propagation problems: computational aspects of 3D energetic Galerkin BEM, Comput. Mech., 51(4), (2013), 475–493.

[3] A. Aimi, L. Desiderio, M. Diligenti, C. Guardasoni, Application of Energetic BEM to 2D Elastodynamic Soft Scattering Problems, Commun. Appl. Ind. Math., 10 (2019), pp. 182–198.

Giulia Di Credico (Innsbruck / Parma)
Fast E-BEM by ACA compression for the resolution of acoustic and elastic 2D exterior problems in time domain

(with Alessandra Aimi and Luca Desiderio) We consider propagation phenomena of acoustic and elastic waves in a 2D unbounded domain, external to a closed set with soft-scattering conditions applied to the boundary. The corresponding differential problems are reformulated in terms of energetic weak space-time Boundary Integral Equations (BIEs) and discretized by means of a Galerkin-type Boundary Element Method (BEM). This Energetic BEM approach (E-BEM) leads to accurate results and allows to overcome the time instabilities arising from the use of standard BEMs [1, 2], however its discretizazion, when Lagrangian basis functions are employed, produces a BEM matrix with a lower triangular Toeplitz block structure composed by temporal blocks that, for growing time, become fully populated. This is a drawback that prevents the application of the considered method to the resolution of large scale realistic problems. Thus, following an accurate theoretical investigation of the low rank nature of these time blocks, we propose a compression technique based on the Adaptive Cross Approximation (ACA) [3], which permits remarkably to reduce the assembly time and memory requirements of the energetic BEM matrix. Moreover, further advances will be presented in the context of fast resolution of acoustic and elastic hard-scattering problems.

[1] A. Aimi, M. Diligenti, C. Guardasoni, I. Mazzieri, S. Panizzi, An energy approach to spacetime Galerkin BEM for wave propagation problems, IJNME, 80 (2009), pp. 1196-1240.

[2] A. Aimi, G. Di Credico, M. Diligenti, C. Guardasoni, Highly accurate quadrature schemes for singular integrals in energetic BEM applied to elastodynamics, JCAM, 410 (2022), 114186.

[3] A. Aimi, L. Desiderio, G. Di Credico, Partially pivoted ACA based acceleration of the Energetic BEM for time-domain acoustic and elastic waves exterior problems, CAMWA, 119 (2022), pp. 351-370.

Joachim Gwinner (München)
Coupling of boundary element and finite element methods with regularization for a nonlinear interface problem with nonmonotone set-valued transmission conditions

In this talk we consider a nonlinear interface problem on an unbounded domain with nonmonotone set-valued transmission conditions. The investigated problem involves a nonlinear monotone partial differential equation in the interior domain and the Laplacian in the exterior domain. Such a scalar interface problem models nonmonotone frictional contact of elastic infinite media. The variational formulation of the interface problem leads to a hemivariational inequality, which lives on the unbounded domain, and so cannot be treated numerically in a direct way. By boundary integral methods the problem is transformed and a novel hemivariational inequality (HVI) is obtained that lives on the interior domain and on the coupling boundary, only. Thus for discretization the coupling of finite elements and boundary elements is the method of choice. In addition smoothing techniques of nondifferentiable optimization are adapted and the nonsmooth part in the HVI is regularized. Thus we reduce the original variational problem to a finite dimensional problem that can be solved by standard optimization tools. We establish convergence results for the total approximation procedure, as well as an asymptotic error estimate for the regularized HVI.

Lothar Banz (Salzburg)
Contact problems in porous media

Biot's poroelasticity describes the deformation of a fluid saturated elastic porous medium due to fluid flow, and the other way around, fluid flow due to deformation of the medium. In this talk we extend the Biot problem by Signorini contact conditions leading to a two field variational inequality of a perturbed saddle point structure. We prove well posedness of the continuous as well as of the hp-finite element discretized problem. By combing the Strang-Lemma with the Falk-Theorem we can prove an a priori error estimate and guaranteed convergence (rates). Moreover, we derive a residual based a posteriori error estimate which is both efficient and reliable. We present several numerical experiments regarding an active set solver and realizable convergence rates of different uniform and adaptive schemes.

Stefan Sauter (Zürich)
The skeleton equation method for acoustic transmission problems with varying coefficients

In this talk we will derive a "skeleton equation method" which transforms a three-dimensional acoustic transmission problem with variable coefficients and mixed boundary conditions to a non-local equation on a two-dimensional skeleton in the domain. For this goal, we introduce and analyse abstract layer potentials as solutions of auxiliary coercive full space transmission problems and derive jump conditions across domain interfaces. This allows us to formulate the non-local skeleton equation as a direct method for the unknown Cauchy data of the original partial differential equation. We develop a theory which inherits coercivity and continuity of the auxiliary full space transmission problem to the resulting variational form of the skeleton equation without relying on an explicit knowledge of Green's function.

Some concrete examples of full and half space transmission problems with piecewise constant coefficients are presented which illustrate the generality of our skeleton equation method and its theory.

Francesco Florian (Zürich)
Discretizing the Skeleton Problem with Boundary Elements

After reformulating the Helmholtz problem as a skeleton variational problem, we show a way to discretize the boundary operators involved. We choose a basis of boundary functions, which allows to discretize both the multi trace and (using a subset) the single trace spaces. Initial numerical tests exhibit the predicted performance and convergence rate.

Abstracts: Saturday

Markus Melenk (Wien)
Local error analysis for the integral fractional Laplacian

(mit Markus Faustmann und Michael Karkulik) For first order discretizations of the integral fractional Laplacian we provide sharp local error estimates on proper subdomains in both the local H^1 -norm and the localized energy norm. Our estimates have the form of a local best approximation error plus a global error measured in a weaker norm.

David Wörgötter (Wien)
Two-Level Error Estimation for the integral fractional Laplacian

We consider an adaptive finite element method for the integral fractional Laplacian of the classical SOLVE-ESTIMATE-MARK-REFINE form. As a-posteriori error indicator, we take a two-level error estimator. Based on certain saturation assumptions, we show reliability and efficiency of the error estimator on meshes generated by newest vertex bisection, which implies linear convergence of the Galerkin error generated by the adaptive algorithm. Moreover, in two space dimensions, we obtain convergence of the algorithm with optimal algebraic rates.

Daniel Seibel (Saarbrücken)
Analytical calculation of matrix entries in Galerkin BEM

In this talk, we present semi-analytical formulae for the calculation of surface integrals appearing in Galerkin boundary element methods. The integrals emerge as the entries of the single and double layer potential and are formed over pairs of surface triangles. Since the integrands are singular if the triangles have non-empty intersection, we apply the Duffy transformation to remove the singularities. We integrate the regularised representations analytically to obtain closed formulae for the cases of identical triangles and triangles with common edge. If the triangles meet in one vertex or are disjoint, we reduce the four-dimensional integrals to one- or two-dimensional integrals respectively. Finally, we verify the correctness and efficiency of the formulae in numerical experiments.

Carsten Carstensen (Berlin)
Towards adaptive Hybrid high-order methods (HHO)

The novel methodology of skeletal schemes led to a new generation of nonstandard discretisations and HHO is one of many of those besides HDG, VEM, DPG, . . . that generalize naturally to nonlinear problems. Can a variational crime lead to discretisations superior to conforming ones? The key for the success of higher-order schemes is through adaptive mesh-refining and the basis of this is a reliable and efficient a posteriori error analysis. The later is a topic in its infancy at least for HHO [5]. While over-stabilization enables some progress for DG and VEM, it is a refined analysis [6] that makes a stabilization-free a posteriori error estimate possible for the HHO [1]. The presentation reports on recent progress for linear problems [1] and then focusses on two very different nonlinear applications with – in comparison to conforming FEM – complementary advantages.
The fine-tuned extra-stabilized direct computation of guaranteed lower eigenvalue bounds allows for optimal convergence rates of a variant of HHO [2]. The appealing robust parameter selection allows the adaptive computation with higher convergence rates in numerical benchmarks.

The class of degenerate convex minimization problems with two-sided growth conditions and an appropriate convexity control [3,4] allows convergent adaptive mesh-refinement for the dual stress-type variable.
The presentation reports on the state of research in joint projects with S. Puttkammer (Berlin) and N.T. Tran (Jena)

[1] F. Bertrand, C. Carstensen, B. Gräßle, N. T. Tran. Stabilization-free HHO a posteriori error control. arXiv:2207.01038

[2] Carstensen, C., Ern, A., and Puttkammer, S. Guaranteed lower bounds on eigenvalues of elliptic operators with a hybrid high-order method. Numer. Math. (2021)

[3] C. Carstensen, N. T. Tran. Unstabilized hybrid high-order method for a class of degenerate convex minimization problems. SINUM (2021)

[4] C. Carstensen, N. T. Tran. Convergent adaptive hybrid higher-order schemes for convex minimization. Numer. Math. (2022)

[5] A. Ern, J.L. Guermond. Finite elements II—Galerkin approximation, elliptic and mixed PDEs, vol. 73. Springer, Cham (2021)

[6] A. Ern, P. Zanotti. A quasi-optimal variant of the hybrid high-order method for elliptic partial differential equations with H^1 loads. IMA J. Numer. Anal. 40(4), 2163-2188 (2020)

[7] D.A. Di Pietro, J. Droniou. The hybrid high-order method for polytopal meshes, vol. 19. Springer, Cham (2020)