

Übungsblatt 10

$$(a) (i) P(A) = P(A|M_1)P(M_1) + P(A|M_2)P(M_2)$$

$$= 0,07 \cdot 0,4 + 0,02 \cdot 0,6 = 0,028 + 0,012 =$$

$$= 0,04$$

$$(ii) P(M_1|A) = \frac{P(A|M_1)P(M_1)}{P(A)} =$$

$$= \frac{0,07 \cdot 0,4}{0,04} = 0,7$$

$$P(M_2|A) = \frac{P(A|M_2)P(M_2)}{P(A)} =$$

$$= \frac{0,02 \cdot 0,6}{0,04} = 0,3$$

$$(b) (i) p(\theta) = \begin{cases} 1 & \theta \in [0,1] \\ 0 & \text{sonst} \end{cases}$$

$$p((\theta|\xi) = (1,1,0,0,0)) = c \cdot \theta^2 (1-\theta)^3 \quad \theta \in [0,1]$$

$$1 = c \int_0^1 \theta^2 (1-\theta)^3 d\theta =$$

$$= c \cdot \int_0^1 (\theta^2 - 3\theta^3 + 3\theta^4 - \theta^5) d\theta$$

$$= c \left(\frac{\theta^3}{3} - \frac{3}{4}\theta^4 + \frac{3}{5}\theta^5 - \frac{1}{6}\theta^6 \right) \Big|_0^1 =$$

$$= c \left(\frac{1}{3} - \frac{3}{4} + \frac{3}{5} - \frac{1}{6} \right) = \frac{c}{60} \Rightarrow c = 60$$

$$\begin{aligned}
 \hat{\theta} &= 60 \int_0^1 \theta \cdot \theta^2 (1-\theta)^3 d\theta = \\
 &= 60 \int_0^1 (\theta^3 - 3\theta^4 + 3\theta^5 - \theta^6) d\theta \\
 &= 60 \cdot \left(\frac{\theta^4}{4} - \frac{3}{5}\theta^5 + \frac{3}{6}\theta^6 - \frac{1}{7}\theta^7 \right) \Big|_0^1 \\
 &= 60 \cdot \left(\frac{1}{4} - \frac{3}{5} + \frac{1}{2} - \frac{1}{7} \right) = \frac{60}{140} = \frac{3}{7} \approx 0,43
 \end{aligned}$$

$$(ii) \quad p((\theta|\xi) = (0,0,1,0,0)) = c \cdot \theta (1-\theta)^4$$

$$\begin{aligned}
 1 &= c \int_0^1 \theta (1-\theta)^4 d\theta = & \varphi = 1-\theta \quad d\varphi = -d\theta \\
 &= -c \int_0^1 (1-\varphi) \varphi^4 d\varphi = c \int_0^1 (\varphi^4 - \varphi^5) d\varphi & \theta = 1-\varphi \\
 &= c \cdot \left(\frac{1}{5} - \frac{1}{6} \right) = \frac{1}{30} \Rightarrow c = 30
 \end{aligned}$$

$$\begin{aligned}
 \hat{\theta} &= 30 \int_0^1 \theta \cdot \theta (1-\theta)^4 d\theta = 30 \int_0^1 (1-\varphi)^2 \varphi^4 d\varphi = \\
 &= 30 \int_0^1 (\varphi^4 - 2\varphi^5 + \varphi^6) d\varphi = 30 \left(\frac{1}{5} - \frac{2}{6} + \frac{1}{7} \right) = \\
 &= \frac{30}{105} = 2/7 \approx 0,2857
 \end{aligned}$$

$$(iii) \quad p((\theta|\xi) = (1,1,1,1,1)) = c \cdot \theta^5$$

$$1 = c \cdot \int_0^1 \theta^5 d\theta = c \cdot \frac{1}{6} \Rightarrow c = 6$$

$$\hat{\theta} = 6 \cdot \int_0^1 \theta \cdot \theta^5 d\theta = 6 \cdot \frac{1}{7} = 0,86$$

$$(c) (i) \quad p(\theta) = \begin{cases} \frac{1}{0.8-0.2} \stackrel{5/3}{=} & \theta \in [0.2, 0.8] \\ 0 & \text{sonst} \end{cases}$$

$\int_0^1 \text{min} \quad \frac{5}{3} \int_{0.2}^{0.8}$

$$1 = c \cdot \frac{5}{3} \int_{0.2}^{0.8} \theta^2 (1-\theta)^3 d\theta = \dots = c \cdot \frac{307}{12500} = c \cdot 0.0246$$

$$\Rightarrow c = 40.716$$

$$\hat{\theta} = 40.716 \cdot \frac{5}{3} \int_{0.2}^{0.8} \theta \cdot \theta^2 (1-\theta)^3 d\theta = \dots = 0.4524$$

$$(ii) \quad p(\theta) = \begin{cases} \frac{1}{0.6-0.4} = 5 & \theta \in [0.4, 0.6] \\ 0 & \text{sonst} \end{cases}$$

$$1 = c \cdot 5 \int_{0.4}^{0.6} \theta^2 (1-\theta)^3 d\theta = \dots = 0.0304$$

$$c = 32.8659$$

$$\hat{\theta} = 32.8659 \cdot 5 \cdot \int_{0.4}^{0.6} \theta \cdot \theta^2 (1-\theta)^3 d\theta =$$

$$\dots = 0.4935$$

$$(iii) \quad p(\theta) = 6\theta(1-\theta) \quad \theta \in [0,1]$$

$$1 = c \int_0^1 \theta^2 (1-\theta)^3 \cdot 6\theta(1-\theta) d\theta =$$

$$= \frac{3}{140} \Rightarrow c = \frac{140}{3}$$

$$\hat{\theta} = \frac{140}{3} \int_0^1 \theta \cdot \theta^2 (1-\theta)^3 \cdot 6\theta(1-\theta) d\theta =$$

$$= \frac{4}{9} = 0.444$$