Übungsblatt 11

(a) (i) $P(A) = P(A|M_1)P(M_1) + P(A|M_2)P(M_2)$

$= 0,97 \cdot 0,4 + 0,02 \cdot 0,6 = 0,028 + 0,012 =$

$= 0,04$

(ii) $P(M_1|A) = \frac{P(A|M_1)P(M_1)}{P(A)} =$

$= \frac{0,07 \cdot 0,4}{0,04} = 0,7$

$P(M_2|A) = \frac{P(A|M_2)P(M_2)}{P(A)} =$

$= \frac{0,02 \cdot 0,6}{0,04} = 0,3$

(b) (i) $p(\theta) = \begin{cases} 1 & \theta \in [0,1] \\ 0 & \text{sonst} \end{cases}$

$p(\theta | \xi = (1,1,0,0,0)) = c \cdot \theta^2 (1-\theta)^3$ $\quad \theta \in [0,1]$

$1 = c \int_0^1 \theta^2 (1-\theta)^3 d\theta =$

$= c \cdot \int_0^1 (\theta^2 - 3\theta^3 + 3\theta^4 - \theta^5) d\theta$

$= c \left(\frac{\theta^3}{3} - \frac{3}{4}\theta^4 + \frac{3}{5}\theta^5 - \frac{1}{6}\theta^6 \right) \Big|_0^1 =$

$= c \left(\frac{1}{3} - \frac{3}{4} + \frac{3}{5} - \frac{1}{6} \right) = \frac{c}{60} \Rightarrow c = 60$

$$\hat{\theta} = 60 \int_0^1 \theta \cdot \theta^2 (1-\theta)^3 \, d\theta =$$

$$= 60 \int_0^1 (\theta^3 - 3\theta^4 + 3\theta^5 - \theta^6) \, d\theta$$

$$= 60 \cdot \left(\frac{\theta^4}{4} - \frac{3}{5}\theta^5 + \frac{3}{6}\theta^6 - \frac{1}{7}\theta^7 \right) \Big|_0^1$$

$$= 60 \cdot \left(\frac{1}{4} - \frac{3}{5} + \frac{1}{2} - \frac{1}{7} \right) = \frac{60}{140} = \frac{3}{7} \approx 0{,}43$$

(ii) $p(\theta \mid \xi = (1,1,1,1,1)) = c \cdot \theta^5$

$$1 = c \cdot \int_0^1 \theta^5 \, d\theta = c \cdot \frac{1}{6} \Rightarrow c = 6$$

$$\hat{\theta} = 6 \cdot \int_0^1 \theta \cdot \theta^5 \, d\theta = 6 \cdot \frac{1}{7} = 0{,}86$$

(c) (i) $p(\theta) = \begin{cases} \frac{1}{0.8-0.2} = 5/3 & \theta \in [0.2, 0.8] \\ 0 & \text{sonst} \end{cases}$

soll $\int_0^1$ sein $\frac{5}{3} \int_{0.2}^{0.8}$

$$1 = c \cdot \frac{5}{3} \int_{0.2}^{0.8} \theta^2 (1-\theta)^3 \, d\theta = \ldots = c \cdot \frac{307}{12500} = c \cdot 0.0246$$

$$\Rightarrow c = 40.716$$

$$\hat{\theta} = 40.716 \cdot \frac{5}{3} \int_{0.2}^{0.8} \theta \cdot \theta^2 (1-\theta)^3 \, d\theta = \ldots = 0{,}4524$$

(ii) $p(\theta) = \begin{cases} \frac{1}{0.6-0.4} = 5 & \theta \in [0.4, 0.6] \\ 0 & \text{cond} \end{cases}$

$$1 = c \cdot 5 \int_{0.4}^{0.6} \theta^2 (1-\theta)^3 d\theta = \ldots = 0.0304$$

$$c = 32.8659$$

$$\hat{\theta} = 32.8659 \cdot 5 \cdot \int_{0.4}^{0.6} \theta \cdot \theta^2 (1-\theta)^3 d\theta =$$

$$\ldots = 0.4935$$