I Intensivstation Ī 16 U 14 G 3

U Ungeimpft

G Geimpft

$$P(G|\bar{I}) \approx \frac{3}{16} \qquad P(U|\bar{I}) \approx \frac{13}{16}$$

$$P(\bar{I}|G)P(G) = P(G|\bar{I})P(\bar{I})$$

$$P(\bar{I}|U)P(U) = P(U|\bar{I})P(\bar{I})$$

$$\frac{P(I|U)}{P(\bar{I}|G)} = \frac{\dfrac{P(U|I)P(\bar{I})}{P(U)}}{\dfrac{P(G|\bar{I})P(\bar{I})}{P(G)}} = \frac{P(U|\bar{I})P(G)}{P(G|\bar{I})P(U)}$$

(i) $P(U|I) = \frac{13}{16} = 0{,}8125$ $P(G|I) = \frac{3}{16} = 0{,}1875$

(ii) Impfquote $0{,}6799$ $0{,}8312$ (iii)

α $9{,}2041$ $21{,}3381$

für Normalstation:

(i) $P(U|N) = \frac{19}{24} = 0{,}7917$ $P(G|N) = \frac{5}{24} = 0{,}2083$

(ii) Impfquote $0{,}6799$ $0{,}8312$ (iii)

α $8{,}0713$ $18{,}7118$

Übungsblatt 10

(4) (i) $p(\theta) = \begin{cases} 1 & \theta \in [0,1] \\ 0 & \text{sonst} \end{cases}$

$p((\theta|\xi) = (1,1,0,0,0)) = c \cdot \theta^2 (1-\theta)^3$ $\quad \theta \in [0,1]$

$1 = c \int_0^1 \theta^2 (1-\theta)^3 d\theta =$

$= c \cdot \int_0^1 (\theta^2 - 3\theta^3 + 3\theta^4 - \theta^5) d\theta$

$= c \left(\frac{\theta^3}{3} - \frac{3}{4}\theta^4 + \frac{3}{5}\theta^5 - \frac{1}{6}\theta^6 \right) \Big|_0^1 =$

$= c \left(\frac{1}{3} - \frac{3}{4} + \frac{3}{5} - \frac{1}{6} \right) = \frac{c}{60} \Rightarrow c = 60$

$$\hat{\theta} = 60 \int_0^1 \theta \cdot \theta^2 (1-\theta)^3 \, d\theta =$$

$$= 60 \int_0^1 (\theta^3 - 3\theta^4 + 3\theta^5 - \theta^6) \, d\theta$$

$$= 60 \cdot \left(\frac{\theta^4}{4} - \frac{3}{5}\theta^5 + \frac{3}{6}\theta^6 - \frac{1}{7}\theta^7 \right) \Big|_0^1$$

$$= 60 \cdot \left(\frac{1}{4} - \frac{3}{5} + \frac{1}{2} - \frac{1}{7} \right) = \frac{60}{140} = \frac{3}{7} \approx 0{,}43$$

(ii) $p((\theta|\xi) = (0,0,1,0,0)) = c\,\theta(1-\theta)^4$

$$1 = c \int_0^1 \theta(1-\theta)^4 \, d\theta = \qquad \varphi = 1-\theta \quad d\varphi = -d\theta \quad \theta = 1-\varphi$$

$$= -c \int (1-\varphi)\varphi^4 \, d\varphi = c \int_0^1 (\varphi^4 - \varphi^5) \, d\varphi$$

$$= c \cdot \left(\frac{1}{5} - \frac{1}{6} \right) = \frac{1}{30} \Rightarrow c = 30$$

$$\hat{\theta} = 30 \int_0^1 \theta \cdot \theta(1-\theta)^4 \, d\theta = 30 \int_0^1 (1-\varphi)^2 \varphi^4 \, d\varphi =$$

$$= 30 \int_0^1 (\varphi^4 - 2\varphi^5 + \varphi^6) \, d\varphi = 30 \left(\frac{1}{5} - \frac{2}{6} + \frac{1}{7} \right) =$$

$$= \frac{30}{105} = 2/7 \approx 0{,}2857$$

(iii) $p((\theta|\xi) = (1,1,1,1,1)) = c \cdot \theta^5$

$$1 = c \cdot \int_0^1 \theta^5 \, d\theta = c \cdot \frac{1}{6} \Rightarrow c = 6$$

$$\hat{\theta} = 6 \cdot \int_0^1 \theta \cdot \theta^5 \, d\theta = 6 \cdot \frac{1}{7} = 0{,}86$$

(c) (i) $p(\theta) = \begin{cases} \frac{1}{0.8-0.2} = 5/3 & \theta \in [0.2, 0.8] \\ 0 & \text{sonst} \end{cases}$

soll $\int_0^1$ sein $\frac{5}{3}\int_{0.2}^{0.8}$

$$1 = c\cdot\frac{5}{3}\int_{0.2}^{0.8}\theta^2(1-\theta)^3 d\theta = \ldots = c\cdot\frac{307}{12500} = c\cdot 0.0246$$

$$\Rightarrow c = 40.716$$

$$\hat{\theta} = 40.716\cdot\frac{5}{3}\int_{0.2}^{0.8}\theta\cdot\theta^2(1-\theta)^3 d\theta = \ldots = 0,4524$$

(ii) $p(\theta) = \begin{cases} \frac{1}{0.6-0.4} = 5 & \theta \in [0.4, 0.6] \\ 0 & \text{sonst} \end{cases}$

$$1 = c\cdot 5\int_{0.4}^{0.6}\theta^2(1-\theta)^3 d\theta = \ldots = 0.0304$$

$$c = 32,8659$$

$$\hat{\theta} = 32.8659\cdot 5\cdot\int_{0.4}^{0.6}\theta\cdot\theta^2(1-\theta)^3 d\theta =$$

$$\ldots = 0.4935$$

(iii) $p(\theta) = 6\theta(1-\theta) \quad \theta \in [0,1]$

$$1 = c \int_0^1 \theta^2 (1-\theta)^3 \cdot 6\theta(1-\theta)\, d\theta =$$

$$= \frac{3}{140} \Rightarrow c = \frac{140}{3}$$

$$\hat{\theta} = \frac{140}{3} \int_0^1 \theta \cdot \theta^2 (1-\theta)^3 \cdot 6\theta(1-\theta)\, d\theta =$$

$$= \frac{4}{9} = 0.444$$